

Useful entanglement from the Pauli principle

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We address the question whether identical-particle entanglement is a useful resource for quantum information processing. We answer this question positively by reporting a scheme to create entanglement using semiconductor quantum wells. The Pauli exclusion principle forces quantum correlations between the spins of two independent fermions in the conduction band. Selective electron-hole recombination then transfers this entanglement to the polarization of emitted photons, which can subsequently be used for quantum information tasks.

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Entanglement is both an essential key for understanding spooky quantum phenomena¹ and a useful resource which allows the implementation of desirable quantum information tasks.² It is then naturally important to develop methods of extracting entanglement from physical systems as well as using it in different applications. The usual ways of creating entanglement include interactions,³ postselection (e.g., in parametric down conversion⁴), or a combination of such strategies.⁵

Quantum correlations naturally appear in fermionic systems. A pure state of a pair of identical fermions is always antisymmetric, hence somehow entangled. This entanglement comes from the indistinguishability of the fermions and can manifest itself in one or more degrees of freedom, depending, for example, on the spatial shape of the wave function.⁶ However, there is an interesting ongoing debate over whether it is possible to use this strictly spin-statistical entanglement to perform quantum information tasks.⁷

In this Brief Report, we present a solution to this question by proposing a scheme to extract entanglement created solely by the Pauli exclusion principle. Our scheme is quite unorthodox in the sense that decoherence, usually viewed as an enemy for entanglement production, actually plays an important role in the extraction of this identical-particle entanglement.

Decoherence can usually be understood as the creation of spurious and unavoidable correlations between the system of interest and its environment.⁸ Such correlations imply information loss and entropy creation, with the state of the system being described by a density operator. When the system in question is composed, decoherence also tends to wash out entanglement. However, one very special situation occurs when decoherence is dictated by a null temperature heat reservoir, in which case the system asymptotically approaches its lowest energy level. Whenever the ground state is nondegenerate, the result of null temperature decoherence is a pure state. For composed systems, if the nondegenerate ground state is also entangled, decoherence can actually create entangled states.

Here, we describe how to combine dissipation and the

Pauli exclusion principle to produce entangled fermions. Then, we show how to use selective recombination to extract this fermionic entanglement as useful propagating photons entangled in polarization. These general ideas apply to different fermionic systems. As an example, we describe, from now on, their application in solid state physics. In fact, recent technological development has placed semiconductors among the systems capable of producing entangled particles. For example, the decay of biexcitons in a quantum dot,^{9,10} or the decay of excitons in two coupled quantum dots,¹¹ has been used to produce entangled photons. However, in none of these works, the Pauli exclusion principle is the central source of the generation of entanglement.

Consider that a semiconductor in its ground state is excited by the promotion of two electrons to the conduction band. These electrons will be described by some quantum state with momentum and spin distribution. Due to phonon scattering in a short time scale, the spin state will be essentially random (supposing no energy difference between the possible spin polarizations). With the condition that the relaxation time τ_D is much shorter than the recombination time τ_{eh} , the electronic momentum distribution will tend to the bottom of the band. In fact, the quantum state will approach the lowest energy state of the band which, due to the Pauli principle, corresponds to the null momentum spin singlet. In a statistically independent way, electrons in the valence band also relax, with the net effect of promoting the existing holes to the top of the band. Again, assuming that the relaxation time of the holes is much shorter than the respective recombination time, there will be a singlet of holes in the top of the valence band as well. So far, quantum correlations have been created by the Pauli principle through relaxation. The remaining question is whether these correlations can be used to implement some quantum protocol.

We obtain a positive answer by considering a selection rule for the radiative electron-hole recombination: electrons with spin $+1/2$ ($-1/2$) can only decay emitting photons circularly polarized to the right (left) [Fig. 1(b)]. After both electrons have decayed, we obtain two photons in a polariza-

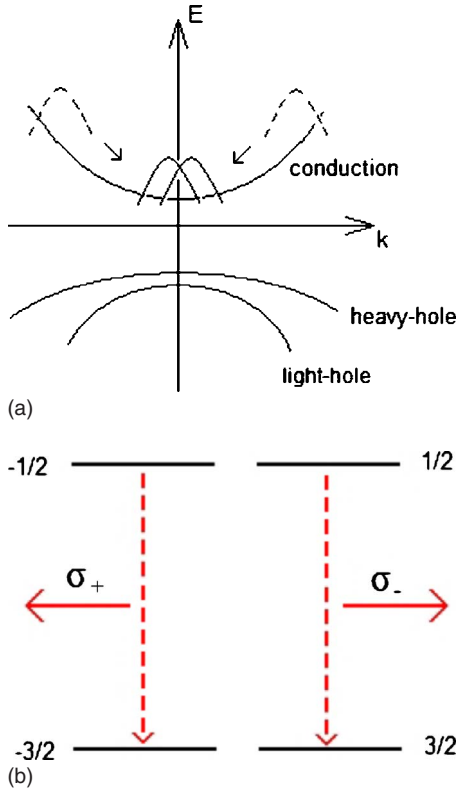


FIG. 1. (Color online) (a) Band structure for the semiconductor quantum well. For each value of k , there can exist just two electrons according to the Pauli principle. (b) Selection rules force the transitions $J_z = -\frac{3}{2} \leftrightarrow J_z = -\frac{1}{2}$ through an emission or absorption of a σ_+ photon and $J_z = \frac{3}{2} \leftrightarrow J_z = \frac{1}{2}$ through an emission or absorption of a σ_- photon.

tion entangled state, which can then be used for different quantum information protocols. Note that the only effect of the recombination is to transfer the fermionic entanglement into the emitted photons.

Quantum wells prepared with semiconductors of the group III-V¹² constitute examples of physical systems that combine all the necessary ingredients. Their conduction (valence) band has orbital angular momentum $L=0$ ($L=1$). Due to the spin-orbit interaction, the valence band splits into heavy holes ($J_z = \pm 3/2$) and light holes ($J_z = \pm 1/2$). The two-dimensional confinement lifts their degeneracy at $k=0$ producing different values for their energy gap and, thus, allowing for the desirable selectivity [see Fig. 1(a)]. At the same time, these systems present appropriate time scales since for semiconductors, typically $\tau_D \sim 10^{-12}$ s and $\tau_{eh} \sim 10^{-9}$ s.

From fermions to photons. Let us now treat in some details the recombination process. Suppose we have a semiconductor quantum well with exactly two excitations with well defined momentum k above the ground state (full valence band). Consider the creation operator of two particles (electron+hole):

$$\Psi_s^\dagger(k) = e_s^\dagger(k)h_s^\dagger(-k), \quad (1)$$

where $e_s^\dagger(k)$ [respectively, $h_s^\dagger(k)$] creates an electron (hole) in the conduction (valence) band with logical spin s and mo-

mentum k . The spin notation is utilized to emphasize correlation in the creation process, and the following correspondence between the real spins with the logical basis is implied through this Brief Report:

	$ 0\rangle$	$ 1\rangle$
Electrons	$-1/2$	$1/2$
Holes	$-3/2$	$3/2$

The spin state we are interested in can be described by a spin density operator for the electrons and holes determined (up to normalization) by the correlation function:⁶

$$\begin{aligned} \rho_{rr's's'} &= \langle \Phi_0 | \Psi_r(k) \Psi_{r'}(k) \Psi_s^\dagger(k) \Psi_{s'}^\dagger(k) | \Phi_0 \rangle \\ &= \langle \phi_0^{(e)} | e_r(k) e_{r'}(k) e_s^\dagger(k) e_{s'}^\dagger(k) | \phi_0^{(e)} \rangle \\ &\quad \times \langle \phi_0^{(h)} | h_r(-k) h_{r'}(-k) h_s^\dagger(-k) h_{s'}^\dagger(-k) | \phi_0^{(h)} \rangle, \quad (2) \end{aligned}$$

where $|\phi_0^{(e)}\rangle(|\phi_0^{(h)}\rangle)$ denotes the electron (hole) initial state (vacuum) and $|\Phi_0\rangle$ is their tensor product. As the operators obey fermionic anticommutation rules

$$[e_s^\dagger(k), e_{s'}(k')]_+ = \delta_{ss'} \delta(k - k') \quad (3)$$

[the same for $h_s(k)$], we have

$$\rho_{rr's's'} = (\delta_{rs'} \delta_{r's} - \delta_{rs} \delta_{r's'})^2. \quad (4)$$

Note that we use a shortened label to represent the matrix elements (2). In the electron-hole-electron-hole ordering, this operator is a density matrix representing the (un-normalized) state

$$|\psi\rangle = \left| -\frac{1}{2}, -\frac{3}{2}; +\frac{1}{2}, +\frac{3}{2} \right\rangle + \left| +\frac{1}{2}, +\frac{3}{2}; -\frac{1}{2}, -\frac{3}{2} \right\rangle. \quad (5)$$

In order to enhance the emission process and also to have control over the emitted photons, the sample can be placed within an optical cavity¹³ in resonance with the transition between the top the heavy-hole valence band and the bottom of the conduction band. Following these selection rules, the process described above will generate photon pairs in the polarization state

$$|\sigma_- \sigma_+\rangle + |\sigma_+ \sigma_-\rangle, \quad (6)$$

which is a maximally entangled Bell state. Note that it is essential that the electrons have the same momentum for the creation of a maximally entangled pair of photons.

Some imperfections. The scenario that was discussed up to now is pretty much idealized. It is important to stress, however, that one very robust point in favor of this proposal is its independency on the specific model of decoherence used. Whenever the conditions on the time scales here imposed are fulfilled, the state of the system before recombination will be very close to the one described here, and so will the state of the emitted photons.

One nice way to mimic the effects of imperfections in our approach is to consider a broadening in the momentum dis-

tribution and also an imperfect coincidence of the momenta. This can be phenomenologically taken into account by modifying Eq. (1) to

$$\Psi_{ss'}^\dagger(k) = \int \int f(k_j - k) f(\tilde{k}_j - \tilde{k}) e_s^\dagger(k_j) h_s^\dagger(\tilde{k}_j) dk_j d\tilde{k}_j. \quad (7)$$

This operator creates an electron with spin s and momentum distribution given by the function $f(k_j - k)$ and a hole with spin s and momentum distribution given by $f(\tilde{k}_j - \tilde{k})$. Later, we will associate $k = -\tilde{k}$. With this operator, Eq. (2) can be rewritten as

$$\begin{aligned} \rho_{rr'ss'} &= \int dk_0 \cdots dk_3 f^*(k_0 - k) f^*(k_1 - k') f(k_2 - k) f(k_3 - k') \\ &\times \langle \phi_0^{(e)} | e_r(k_0) e_{r'}(k_1) e_s^\dagger(k_2) e_s^\dagger(k_3) | \phi_0^{(e)} \rangle \\ &\times \int d\tilde{k}_0 \cdots d\tilde{k}_3 f^*(\tilde{k}_0 - \tilde{k}) f^*(\tilde{k}_1 - \tilde{k}') f(\tilde{k}_2 - \tilde{k}) \\ &\times f(\tilde{k}_3 - \tilde{k}') \langle \phi_0^{(h)} | h_r(\tilde{k}_0) h_{r'}(\tilde{k}_1) h_s^\dagger(\tilde{k}_2) h_s^\dagger(\tilde{k}_3) | \phi_0^{(h)} \rangle. \end{aligned} \quad (8)$$

Anticommutation rules (3) imply that the only non-null matrix elements are

$$\rho_{0000} = \rho_{1111} = [L(k, k') - M(k, k')][\tilde{L}(\tilde{k}, \tilde{k}') - \tilde{M}(\tilde{k}, \tilde{k}')], \quad (9a)$$

$$\rho_{0101} = \rho_{1010} = L(k, k') \tilde{L}(\tilde{k}, \tilde{k}'), \quad (9b)$$

$$\rho_{0110} = \rho_{1001} = M(k, k') \tilde{M}(\tilde{k}, \tilde{k}'), \quad (9c)$$

where

$$L(k, k') = \int dk_0 dk_1 |f(k_0 - k)|^2 |f(k_1 - k')|^2, \quad (10a)$$

$$M(k, k') = \int dk_0 dk_1 f^*(k_0 - k) f^*(k_1 - k') f(k_1 - k) f(k_0 - k'), \quad (10b)$$

with similar expressions for $\tilde{L}(\tilde{k}, \tilde{k}')$ and $\tilde{M}(\tilde{k}, \tilde{k}')$ by changing $k \mapsto \tilde{k}$ and $k' \mapsto \tilde{k}'$.

The emitted photon (non-normalized) polarization state will thus be

$$\rho = \begin{pmatrix} (L - M)(\tilde{L} - \tilde{M}) & & & & \\ & \tilde{L}\tilde{L} & \tilde{M}\tilde{M} & & \\ & \tilde{M}\tilde{M} & \tilde{L}\tilde{L} & & \\ & & & (L - M)(\tilde{L} - \tilde{M}) & \\ & & & & \end{pmatrix}, \quad (11)$$

where we leave blank the null entries. Note that when $k = k'$ and $\tilde{k} = \tilde{k}'$ hold, $L = M$ and $\tilde{L} = \tilde{M}$, giving a maximally

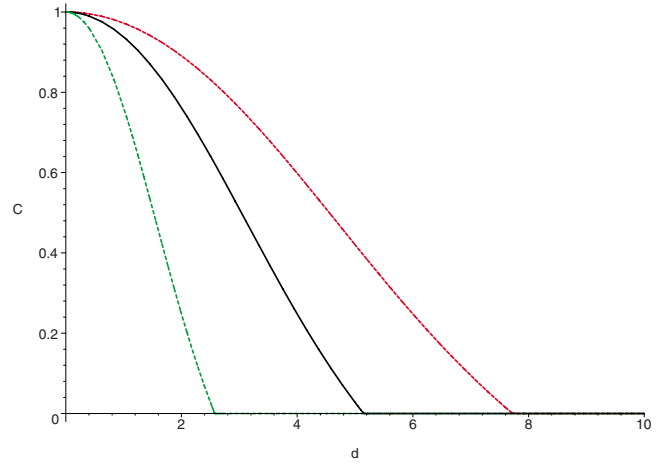


FIG. 2. (Color online) Photonic entanglement versus $d = |k - k'|$. Green dotted line, $\delta = 2$; black solid line, $\delta = 4$; red dashed curve, $\delta = 6$, where δ is a measure of the width of the momentum distribution, assumed the same for electrons and holes [see Eq. (12)]. We see that the greater the spread in the momentum, the higher the entanglement between the photons. This is due to the fact that wider momentum distributions blur the difference in k 's, so that once again, it is impossible to distinguish the electrons by momentum.

entangled photonic state. Notice the generality of this result: state Eq. (11) is written in terms of arbitrary momentum distributions of the electrons before the decaying process. For illustration, a chart of the entanglement between the two photons (characterized by the concurrence¹⁴) versus the difference in momentum distribution of the decaying electrons is displayed in Fig. 2. We have chosen a Lorentzian distribution of spread δ centered in k (k') for the momentum distributions, i.e.,

$$f(k_j - k) = \frac{\delta}{\pi[(k_j - k)^2 + \delta^2]}. \quad (12)$$

Conclusion. From the fundamental point of view, we stress once again that the origin of the entanglement lies in the fermionic nature of the electrons. We claim that this gives a decisive positive answer to the question whether identical-particle entanglement is useful for quantum information purposes. Specifically, identical-particle entanglement can, in fact, be extracted and converted into usual entanglement, and one important ingredient in this convertibility is the use of more than one degree of freedom. Finally, we would like to emphasize again the role played by the coupling to the environment. Usually, decoherence is seen as the road from quantum to classical, which makes it a plague for quantum information tasks. Here, it plays the crucial role of washing out the distinguishable origin of the input photons, allowing the extraction of entanglement from the Pauli principle.

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