

Simultaneous analytical characterisation of two unknown ultrashort laser pulses: blind-MEFISTO

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Abstract: We present a new analytical methodology that allows simultaneously characterizing two unknown ultrashort pulses with the same central frequency. This analysis is based on Fourier transform spectrally resolved interferometric collinear correlations in the delay domain.

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Previously, the simultaneous characterization of two unknown ultrashort laser pulses has been carried out using XFROG [1]. In XFROG the non-collinear, spectrally resolved cross-correlation trace is passed through a retrieval algorithm [2] to obtain the pulse information of both pulses. In this paper we will describe a new technique that allows, for the first time, an analytical determination of the complex electric fields of two different pulses which have the same central frequency. This is done by performing a spectrally resolved analysis, in the Fourier domain, of a collinear cross-correlation. We call this technique “blind Measurement of Electric Field by Interferometric Spectral Trace Observation” (blind-MEFISTO).

The blind-MEFISTO technique is equivalent to performing a collinear blind-XFROG measurement but crucially it does not require an iterative retrieval algorithm to extract the pulse information. Blind-MEFISTO can be mathematically described as:

$$I^{SHG}(f, \tau) = \left| F_t \left\{ \left(E(t) \exp[i 2\pi f t] + G(t - \tau) \exp[i 2\pi f_2(t - \tau)] \right)^2 \right\} \right|^2 \quad (1)$$

Here $E(t)$ and $G(t)$ are the slowly varying amplitude of the complex electric field centred at the frequencies f_1 and f_2 . The Fourier transform with respect to the variable t is indicated by F_t . In blind-MEFISTO, we are restrict to the case in which the central frequencies of $E(t)$ and $G(t)$ are equal ($f_1 = f_2 = f_0$). Under this condition phase matching is achieved for all the cross-terms in (1). An example of a generated trace can be seen in figure 1 (a). As we will show, with the new information carried on these terms it will be possible to analytically obtain $E(t)$ and $G(t)$ simultaneously (blind-correlation).

In order to do this, we first calculate the Fourier transform of equation (1) in the τ axis, i.e., $Y^{SHG}(f, \kappa) = F_\tau \{ I^{SHG}(f, \tau) \}$. This expression results in 5 main spectral components (see Fig. 1(b)) that are at frequencies DC , $\pm f_0$ and $\pm 2f_0$. The negative spectral components are the complex conjugate of the positive ones thus, to analyze the information within the transformed trace, we need only to look at the positive frequency components. Every component gives different types of information about the pulses. For blind-MEFISTO we will focus upon the f_0 term however towards the end we will touch upon the use of the other components.

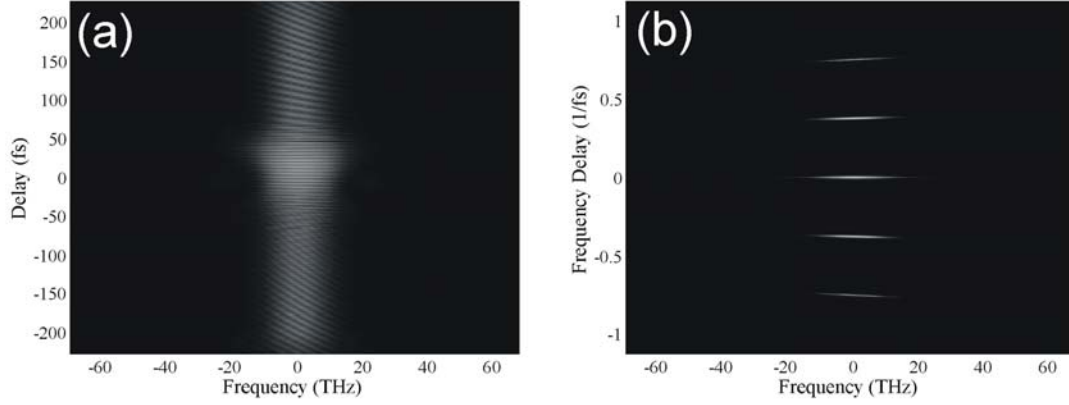


Fig. 1. a) Frequency resolved collinear cross-correlation of two unknown pulses. b) Same trace in the Fourier domain showing its Fourier components at frequencies DC, $\pm f_0$ and $\pm 2f_0$. (For clarity, intensity scale is not linear).

I. - $\kappa = f_0$ component (blind-MEFISTO).

We will start on the components near $\kappa = f_0$. We can write this spectral component as,

$$Y_{\kappa=f_0}^{SHG}(f, \kappa) = 2E_{SHG}(f)E^*(f + f_0 - \kappa)G^*(\kappa - f_0) + 2G_{SHG}^*(f)G(f + f_0 - \kappa)E(\kappa - f) \quad (2)$$

where the second harmonic of the unknown fields, $E_{SHG}(f)$ and $G_{SHG}(f)$, are related with the fundamental pulse following $F_{SHG} = [df'F(f')F(f-f')]$. In what follows, we will show that, using equation (2), it is possible to simultaneously determine $E(f)$ and $G(f)$ in an analytical way (i.e., blind-MEFISTO). To show this, we write all the involved complex magnitudes in polar form, i.e., $Y^{SHG}(f, \kappa) = R(f, \kappa)\exp[i\theta(f, \kappa)]$, $E(f) = U(f)\exp[i\varphi(f)]$ and $G(f) = V(f)\exp[i\gamma(f)]$. Equation (3) can thus be rewritten as

$$\begin{aligned} R(f, \kappa) = & 2U_{SHG}(f)U(f + f_0 - \kappa)V(\kappa - f_0) \times \\ & \exp[\varphi_{SHG}(f) - \varphi(f + f_0 - \kappa) - \gamma(\kappa - f_0) - \theta(f, \kappa)] + \\ & 2V_{SHG}(f)V(f + f_0 - \kappa)U(\kappa - f_0) \times \\ & \exp[-\gamma_{SHG}(f) + \gamma(f + f_0 - \kappa) + \varphi(\kappa - f_0) - \theta(f, \kappa)] \end{aligned} \quad (3)$$

Here we notice that the amplitude of the fundamental [$U(f)$ and $V(f)$] and second harmonic [$U_{SHG}(f)$ and $V_{SHG}(f)$] pulses can be measured but the phases of these pulses [i.e. $\varphi(f)$, $\gamma(f)$, $\varphi_{SHG}(f)$ and $\gamma_{SHG}(f)$] are unknown. To get the phase information we first take two different slices at $\kappa = f_0$ and $\kappa = f_0 - \Delta f$. Then, by taking the real and imaginary parts in equation (3) it is possible to isolate the phase components to obtain:

$$\varphi_{SHG}(f) - \varphi(f) - \gamma(0) = \pm \cos^{-1}[\Omega_1(f, \kappa = f_0)] + \theta(f, \kappa = f_0) \quad 4(a)$$

$$\gamma_{SHG}(f) - \gamma(f) - \varphi(0) = \pm \cos^{-1}[\Omega_2(f, \kappa = f_0)] - \theta(f, \kappa = f_0) \quad 4(b)$$

$$\varphi_{SHG}(f) - \varphi(f + \Delta f) - \gamma(-\Delta f) = \pm \cos^{-1}[\Omega_1(f, \kappa = f_0 - \Delta f)] + \theta(f, \kappa = f_0 - \Delta f) \quad 4(c)$$

$$\gamma_{SHG}(f) - \gamma(f + \Delta f) - \varphi(-\Delta f) = \pm \cos^{-1}[\Omega_2(f, \kappa = f_0 - \Delta f)] - \theta(f, \kappa = f_0 - \Delta f) \quad 4(d)$$

where we have defined,

$$\Omega_1(f, \kappa) = \frac{R^2(f, \kappa) + 4U_{SHG}^2(f)U^2(f + f_0 - \kappa)V^2(\kappa - f_0) - 4V_{SHG}^2(f)V^2(f + f_0 - \kappa)U^2(\kappa - f_0)}{4R(f, \kappa)U_{SHG}(f)U(f + f_0 - \kappa)V(\kappa - f_0)}$$

and

$$\Omega_2(f, \kappa) = \frac{R^2(f, \kappa) - 4U_{SHG}^2(f)U^2(f + f_0 - \kappa)V^2(\kappa - f_0) + 4V_{SHG}^2(f)V^2(f + f_0 - \kappa)U^2(\kappa - f_0)}{4R(f, \kappa)V_{SHG}(f)V(f + f_0 - \kappa)U(\kappa - f_0)}$$

Note that all the functions in the parameters $\Omega_1(f, \kappa)$ and $\Omega_2(f, \kappa)$ can be experimentally obtained. Then, by subtracting equations 4(a) and 4(c) we get

$$\Delta\varphi(f) = \varphi(f + \Delta f) - \varphi(f) = \pm \cos^{-1}[\Omega_1(f, \kappa = f_0)] \mp \cos^{-1}[\Omega_1(f, \kappa = f_0 - \Delta f)] + \theta(f, \kappa = f_0) - \theta(f, \kappa = f_0 - \Delta f) + \gamma(0) - \gamma(-\Delta f) \quad (5)$$

and by subtracting equations 4(b) and 4(d) we obtain

$$\Delta\gamma(f) = \gamma(f + \Delta f) - \gamma(f) = \pm \cos^{-1}[\Omega_2(f, \kappa = f_0)] \mp \cos^{-1}[\Omega_2(f, \kappa = f_0 - \Delta f)] - \theta(f, \kappa = f_0) + \theta(f, \kappa = f_0 - \Delta f) + \phi(0) - \phi(-\Delta f) \quad (6)$$

Equation (5) and (6) are the main result of this work. They allow the characterization of the two pulses by determining the phase of $E(f)$ and $G(f)$ by taking an arbitrary origin $\varphi(0)$ and $\gamma(0)$ and by varying f .

We have numerically tested equation (5) and (6) with two arbitrary pulses, and found perfect agreement with the input pulses. We have, however, found some aspects that are worth noting. Firstly, the terms $\varphi(-\Delta f)$ and $\gamma(-\Delta f)$ in equations (5) and (6) are constants that can be decided arbitrarily. These terms add a linear phase shift that is equivalent to determining the electric field origin in time. Additionally the indetermination in the sign of the function $\cos^{-1}(\Omega)$ results in two solutions that are equivalent to the $E(f)$ and $E^*(f)$ characteristic ambiguity that appears in FROG measurements based on quadratic nonlinearities. Finally, we want to point out that the blind-MEFISTO technique, due of its analytical nature, does not present the nontrivial ambiguities associated with blind-FROG measurements, where a XFROG trace can result in different solutions [3].

II.- DC component (blind cross-FROG).

This term includes the standard cross-FROG trace that has been extensively used to characterize ultra short laser pulses. In a previous paper we outlined a filtering procedure capable to extract this FROG term (in a degenerate case) from the DC components, technique referred to us as CFROG [4]. This allows for a simpler collinear experimental arrangement but it again relies upon the use of retrieval algorithms.

III.- $\kappa = 2f_0$ component (cross-FROG)

The spectral component at $\kappa \approx 2f_0$ can be used to retrieve $E(t)$ when $G(t)$ is known. This term can be written as

$$Y_{\kappa \approx 2f_0}^{SHG}(f, \kappa) = E_{SHG}(f)G_{SHG}^*(f)\delta(f - \kappa + 2f_0) \quad (7)$$

This equation allows an alternative and faster procedure to the one using equations (5) and (6) to analytically determine $E(f)$ when $G(f)$ is the known gating function (cross-correlation).

In conclusion we have outlined a procedure that allows the complex amplitude of two unknown ultrashort pulses to be obtained simultaneously. This is, to the best of our knowledge, the first analytical methodology capable to characterize two unknown ultrashort pulses simultaneously. The technique relies on Fourier analysis after obtaining a spectrally resolved interferometric correlation trace. The blind-MEFISTO methodology has the crucial advantage in that it enables a simple extraction of pulse information without the need of an iterative retrieval algorithm and without having some of the ambiguities that are present in other techniques. Furthermore, it maintains the powerful error checking capabilities that are associated with time-frequency techniques.

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