Diffraction management of focused light beams in optical lattices with a quadratic frequency modulation

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Abstract: We reveal that the effective diffraction experienced by light beams launched along the central guiding channel of optical lattice with a quadratic frequency modulation can be tuned in strength and sign. Complete suppression of the linear diffraction in the broad band of spatial frequencies is shown to be possible, thus profoundly affecting properties of nonlinear self-sustained beams as well. In particular, we report on the properties of a new class of stable solitons supported by such lattices in defocusing media.

OCIS codes: (190.0190) Nonlinear optics; (190.5530) Pulse propagation and solitons

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Diffraction inevitably broadens all finite light beams propagating in the homogeneous linear medium, but a transverse modulation of the refractive index can drastically affect this process. When transverse refractive index modulation is periodic and strong enough the homogeneous diffraction is replaced with the discrete diffraction arising because of the coupling between nearest neighbors in the array [1]. Because the diffraction coefficient for tilted waves depends on their propagation angles, launching of broad laser beam at suitable angles relative to the array axis allows engineering its diffraction broadening [2,3]. This phenomenon was predicted theoretically and experimentally observed in both one- and two-dimensional waveguide arrays [2,4,5], as well as in harmonic refractive index gratings [6]. The interesting phenomenon of diffraction suppression, or grating-mediated waveguiding, was recently reported in Ref. [7]. It is worth noticing that diffraction control of relatively broad laser beams does not require deep refractive index modulations. This fact offers the unique opportunity to use optically induced periodic waveguide arrays, or optical lattices, with the flexibly controlled parameters [8-13]. Besides modification of diffraction properties of light beams, the transverse refractive index modulations drastically affect properties of guided modes supported in the nonlinear regime [8-15]. Thus, the existence domain of lattice solitons is dictated by the Bloch wave spectrum for the lattice.

However the challenging problem of diffraction management is still far from its complete solution. In particular, it is quite important to develop an effective approach to the broadband diffraction control that also allows suppression of diffraction for narrow beams, thus having a broad spatial spectrum. In this work we explore the prospects offered by lattice with quadratic frequency modulation for diffraction control and show that by a properly changing the lattice depth it is possible to achieve broadband diffraction suppression or reversal of diffraction sign for narrow beams launched in the central channel of the lattice parallel to the guiding lattice channels. We also have found that such diffractive properties alter qualitatively the properties of lattice solitons, especially those supported by defocusing media.

We consider propagation of optical radiation along the ξ axis in cubic nonlinear medium with modulation of linear refractive index along transverse η axis, described by the nonlinear Schrödinger equation for dimensionless complex field amplitude q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - \sigma |q|^2 q - pR(\eta)q. \quad (1)$$

Here the longitudinal ξ and transverse η coordinates are scaled to the diffraction length and the input beam width, respectively. The parameter p is proportional to the depth of refractive index modulation, while the function $R(\eta) = \cos[\Omega\eta(1+\alpha\eta^2)]$ stands for the transverse profile of refractive index, where Ω is the carrying spatial frequency and parameter $0 \le \alpha < 1$ characterizes the frequency modulation rate. Parameter $\sigma = \pm 1$ defines the nonlinearity sign (focusing/defocusing). Further we assume that the refractive index modulation depth is small compared to the unperturbed index and is of the order of the nonlinear contribution. It should be pointed out that one can use either technique of Fourier-transform synthesis or computer generated holograms for optical induction of the desired lattice profiles as well as their direct technological fabrication. Notice that Eq. (1) conserves the total energy flow U and Hamiltonian H

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$$U = \int_{-\infty}^{\infty} |q|^2 \, d\eta,$$
$$H = \frac{1}{2} \int_{-\infty}^{\infty} (|\partial q / \partial \eta|^2 - 2pR(\eta)|q|^2 - \sigma |q|^4) d\eta. \tag{2}$$

In the linear limit $(\sigma = 0)$ Eq. (1) can be reduced to the following equation

$$i \frac{dq_k}{d\xi} = \frac{1}{2} k^2 q_k - p \int_{-\infty}^{\infty} R_k(k - k') q_k(k') dk', \tag{3}$$

for the spatial Fourier spectrum of the beam, where

$$q_k = (2\pi)^{-1} \int_{-\infty}^{\infty} q(\eta, \xi) \exp(-ik\eta) d\eta, \quad R_k = (2\pi)^{-1} \int_{-\infty}^{\infty} R(\eta) \exp(-ik\eta) d\eta.$$

Further we consider propagation of symmetric collimated beam launched along ξ -axis in the central channel of the lattice at $\eta = 0$. The possibility of broadband diffraction control of such beams in the lattice with a quadratic frequency modulation (FM) follows from analysis of dispersion relations. The dispersion relations can be obtained explicitly upon substitution of spectrum for the symmetric pairs of plane waves $q(\eta, \xi) = [\exp(ik\eta) + \exp(-ik\eta)] \exp(ib\xi)$ having frequencies $\pm k$ and propagation constants b into Eq. (3):

$$b(k) = -k^2 / 2 + 2pR_k(0) + 2pR_k(2k). \tag{4}$$

The first term in the right side of Eq. (4) stands for the "positive" diffraction (convex phase-front) experienced by waves propagating in uniform media, the second one is a phase shift independent on k, and the last one describes the impact of the lattice due to the Bragg-type scattering. The sign of diffraction and its strength is dictated by the quantity $d^2 b / dk^2$ [2,6]. Notice that Bragg-type scattering is well studied in the context of nonlinear pulse reflection in chirped fiber gratings [16,17]. Thus, pulse reflection in chirped gratings assumes synchronous energy transfer from spectral components of the incident wave-packet to spectral components of Bragg-reflected wave-packet, while a key feature of diffraction suppression that we address here is the broadband phasing of spectral components without energy exchange between them. It is the Bragg-type interaction between counter-propagating waves with equal amplitudes that results in appearance of phase terms $2pR_k(0) + 2pR_k(2k)$ in Eq. (4).

The spatial spectrum of the lattice with a quadratic FM is given by the expression

$$R_k(k) = (12\alpha\Omega)^{-1/2} (\mathrm{Ai}[(3\alpha\Omega)^{-1/3}(\Omega + k)] + \mathrm{Ai}[(3\alpha\Omega)^{-1/3}(\Omega - k)]), \tag{5}$$

where Ai is the Airy function. This expression is depicted in Fig. 1(a). At $\alpha \ll 1$ a first maximum of $R_k(k)$ is achieved at $k \approx \Omega$, and it shifts to the high-frequency region with increase of FM rate α. Notice almost parabolic behavior of $R_k(k)$ dependence in a frequency interval $\Omega > k > -\Omega$. The dispersion curves $b(k)$ are depicted in Fig. 1(b) for different values of the refractive index modulation depth p.

The central result of this paper is that tuning the depth of the lattice affords principal modifications of the diffraction coefficient $d^2 b / dk^2$, including reversal of its sign, for a broad frequency band $k \in [-\Omega, \Omega]$. The critical lattice depth that corresponds to the case of zero diffraction, i.e., $d^2 b / dk^2 = 0$, is given by $p_{\mathrm{cr}} = \{8\Omega(3\alpha\Omega)^{-3/2} \mathrm{Ai}[\Omega(3\alpha\Omega)^{-1/3}]\}^{-1}$ and is depicted in Fig. 1(c) for different α. For $p > p_{\mathrm{cr}}$ the "positive" diffraction is replaced by the

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"negative" one. In the spatial domain this is accompanied by a qualitative change of the phase front shape from convex to concave. Notice, that the possibility to control diffraction sign in the broad frequency band $[-\Omega,\Omega]$ with FM lattices is advantageous in comparison with the diffraction management in harmonic lattices, where zero diffraction can be achieved only for

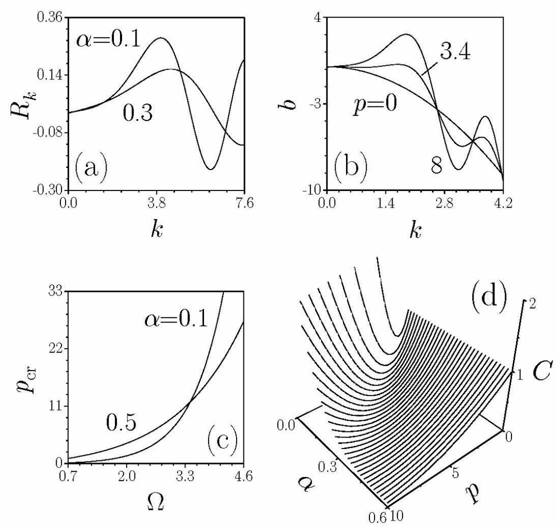

Fig. 1. (a) Spatial spectrum of FM lattice at $\Omega = 3$. (b) Dispersion curves at $\Omega = 3$ and $\alpha = 0.1$. (c) Critical lattice depth versus lattice frequency. (d) Integral factor C on (p,α) plane at $\Omega = 3$.

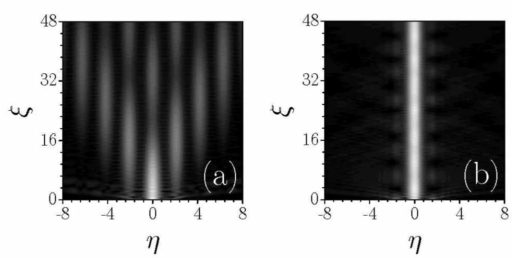

Fig. 2. Beam propagation dynamics in (a) harmonic lattice and (b) lattice with FM rate $\alpha = 0.11$. Lattice depth $p = 3.4$ and frequency $\Omega = 3$. Linear medium.

one particular frequency k and, therefore, diffraction can be suppressed only for broad beams with the correspondingly narrow spectrum.

The efficiency of diffraction suppression for a laser beam with a spectral intensity $S(k)$ can be characterized by the integral factor

$$C(\alpha,p,\Omega) = \int_{-\infty}^{\infty} b^2(k)S(k)dk. \tag{6}$$

This factor is depicted in Fig. 1(d) for the case of input beam $q(\eta,\xi=0) = q_0 \operatorname{sech}(\eta)$, with $S(k) = S_0 \operatorname{sech}^2(\pi k/2)$. The existence of parameter range where diffraction is remarkably suppressed $(C \ll 1)$ is readily apparent. These analytical results are confirmed by the direct numerical integration of Eq. (1). The propagation of such beam in the harmonic lattice is

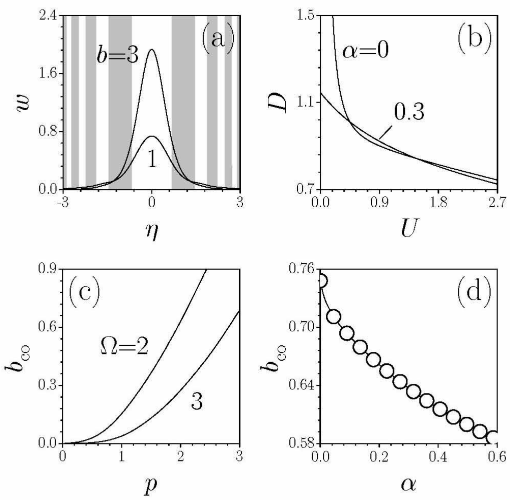

Fig. 3. (a) Profiles of solitons supported by FM lattice at $p = 2$, $\Omega = 2$, $\alpha = 0.3$. (b) Soliton width versus energy flow at $p = 2$, $\Omega = 2$. (c) Propagation constant cutoff versus lattice depth at $\alpha = 0.3$. (d) Propagation constant cutoff versus frequency modulation rate at $p = 2$, $\Omega = 2$. Focusing medium.

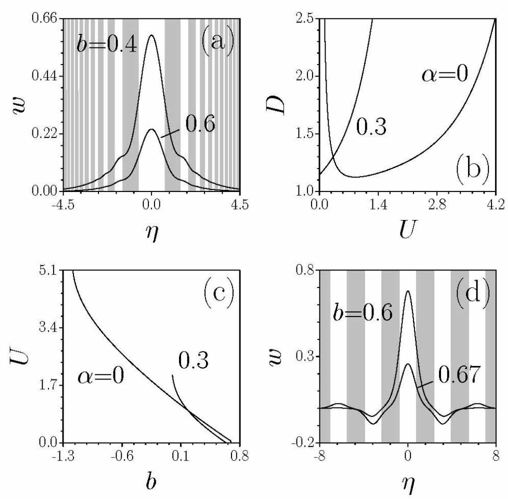

Fig. 4. (a) Profiles of solitons supported by FM lattice at $\alpha = 0.3$. (b) Soliton width versus energy flow. (c) Energy flow versus propagation constant. (d) Profiles of solitons supported by harmonic lattice. In all cases $p = 2$, $\Omega = 2$. Defocusing medium.

accompanied by strong diffraction (Fig. 2(a)), while in the lattice with a suitable quadratic FM the similar beam experiences only small reshaping. Thus, quadratic modulation of the lattice frequency offers a powerful tool for diffraction control of focused light beams.

The specific linear diffractive properties bring several new features into the properties of solitons supported by lattices with quadratic FM whose centers are located at $\eta = 0$. It is especially important in the case of focusing media where reduced diffraction requires smaller light intensities for its compensation. We have found soliton profiles from Eq. (1) in the form $q(\eta, \xi) = w(\eta)\exp(ib\xi)$, where $w(\eta)$ is the real function and b is the propagation constant.

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The properties of the ground-state soliton solutions supported by FM lattice in focusing media $(\sigma = 1)$ are summarized in Fig. 3. With growth of the propagation constant, the soliton gets narrower, while its peak amplitude increases (Fig. 3(a)). There exists a lower cutoff on propagation constant for soliton existence. Close to this cutoff, the soliton transforms into a linear guided mode. The presence of linear localized guided modes even for $\alpha \ll 1$ clearly distinguishes lattices with quadratic FM from their harmonic counterparts that can support in the linear regime only periodic Bloch waves. Consequently, the width D of soliton supported by FM lattice does not diverge when its energy flow $U \to 0$, as it occurs in the harmonic lattices (Fig. 3(b)). The propagation constant cutoff b_{co} increases monotonically with growth of lattice depth p, and drops off with increase of lattice frequency Ω (Fig. 3(c)). Growth of α causes only slight diminishing of cutoff value. A comprehensive linear stability analysis revealed that solitons supported by FM lattices with focusing nonlinearity are stable in the entire domain of their existence.

In the case of defocusing media $(\sigma = -1)$, we discovered that lattices with quadratic FM can support a specific type of solitons that have no analogs in harmonic lattices. Their properties are summarized in Fig. 4. In contrast to the usual gap solitons (Fig. 4(d)) the field of soliton supported by FM lattice does not change its sign (Fig. 4(a)). Such solitons exist in the propagation constants interval $b \in (0, b_{\text{co}})$, where upper cutoff b_{co} for existence coincides with lower cutoff for solitons in focusing media. At $b \to b_{\text{co}}$ such solitons also transform into linear guided modes of FM lattice, thus their width remains finite in contrast to gap solitons that become delocalized in both lower and upper cutoffs. At $b \to 0$ the soliton width diverges as shown in Fig. 4(b) and it acquires the form of a bright peak superimposed on the pedestal. The energy flow is a monotonically decreasing function of propagation constant (Fig. 4(c)). Linear stability analysis showed that solitons supported by defocusing FM lattices are stable in the entire domain of their existence. This result emphasizes the potential of the concept put forward with the quadratic FM lattices – the competition between a negative diffraction and defocusing nonlinearity can result in formation of new types of stable solitons.

We thus conclude stressing that optical lattices with the quadratic frequency modulation afford important new opportunities to control the rate of diffraction spreading and the sign of diffraction for focused light beams launched parallel to the guiding channels in the center of the lattice, even for beams featuring broad transverse spatial spectrum. The new engineerable diffractive properties of such lattices substantially affect not only propagation of linear waves, but also the shape and features of solitons supported by the lattice.

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