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Testing quantum dynamics using signaling

Aditi Sen(De) and Ujjwal Sen
ICFO-Institut de Ciències Fotòniques, Jordi Girona 29, Edifici Nexus II, E-08034 Barcelona, Spain
and Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany

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We consider a physical system in which the description of states and measurements follow the usual quantum mechanical rules. We also assume that the dynamics is linear, but may not be fully quantum (i.e., unitary). We show that in such a physical system, certain *complementary* evolutions, namely, cloning and deleting operations that give a better fidelity than quantum mechanically allowed ones, in one (inaccessible) region, lead to signaling to a far-apart (accessible) region. To show such signaling, one requires certain two-party quantum correlated states shared between the two regions. Subsequent measurements are performed only in the accessible part to detect such phenomenon.

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The existence of quantum correlation in states shared between distant partners has several important fundamental and practical impacts [1]. One can obtain violation of local realism by using states with quantum correlation [2]. On the other hand, one may use states with quantum correlation in nonclassical tasks like cryptography [3], dense coding [4], teleportation [5], etc.

In this paper, we show that quantum correlations can be used to check for or to provide bounds on possible non-quantum effects. Non-quantum effects have generally been divided into two categories: Ones that are nonquantum in the "statics" part of the theory, and ones that are so in the "dynamics" part (see, e.g., [6]). unitary dynamics). We consider a physical system in which (i) *the states* ($|\psi\rangle$, $|\phi\rangle$, etc.) *are elements of a complex Hilbert space*, just as in quantum mechanics, and (ii) *measurements are also assumed just as in quantum mechanics*. The duo is said to form the "statics" part of the theory. We further assume that (iii) *the dynamics is linear*, i.e., $|\psi\rangle \rightarrow |\psi'\rangle$ and $|\phi\rangle \rightarrow |\phi'\rangle$ implies $a|\psi\rangle + b|\phi\rangle \rightarrow a|\psi'\rangle + b|\phi'\rangle$, for complex a and b. Note that (i), (ii), and (iii), by themselves, do not imply a quantum dynamics (i.e., the usual unitary dynamics). For our purposes, it is important to note that (i) and (ii) lead to quantum correlation in states of separated parties. We show that in such a physical system, certain *complementary* families of non-quantum evolutions give rise to signaling. This gives us an independent basis to believe in the quantum dynamics.

In checking for the effect, we will use cloning [7] and deleting [8] operations as our tools. It was shown in [9] that exact cloning or exact deleting results in a change of von Neumann entropy [10]. Within the quantum formalism, although exact cloning and deleting are not possible, approximate versions of such operations *are* possible (see, e.g., [11,12]). To check the effect, one needs to prepare certain bipartite states, which we show to be available within the reach of current technology. Importantly, we do not need to directly observe (perform measurements in) the region whose dynamics is being probed. We suppose that one part (B) of the bipartite state is lost to the "environment." The other part (A) remains in the "accessible" part of the experiment (see Fig. 2). In this paper we show, that in our physical system [i.e., one that follows (i), (ii), and (iii)], whenever the evolution in the *environment* (B), is such that a cloning or deleting happens with a better fidelity than the best quantum-mechanical cloning or deleting machine, there occurs a change of entropy in the *accessible* part (A) of the experiment. This change of entropy can be detected in the A part, and therefore results in a signaling to the A part. Note here that if we believe that signaling is not possible [13], then our results prove that cloning and deleting (which are better than what can be done by the best quantum mechanical machines) are not possible, without assuming the whole quantum dynamics. The reason for the choice of the two operations of cloning and deleting is that it has been generally argued that they are in a sense complementary. Thus, it is conceivable that at least one of such non-quantum-mechanical operations or "nearby" ones are possible to occur, *if at all*, in the environment.

Cloning and deleting. Let us first briefly consider the notions of cloning and deleting. In cloning, we want to have the evolution $|\psi\rangle|0\rangle \rightarrow |\Psi\rangle$, $|\phi\rangle|0\rangle \rightarrow |\Phi\rangle$, where $|0\rangle$ is a fixed "blank" state in which the cloned state is to appear. In the exact case, we want to have $|\Psi\rangle = |\psi\rangle|\psi\rangle$, and $|\Phi\rangle = |\phi\rangle|\phi\rangle$. This, however, is not possible under a quantum-mechanical evolution, when $|\psi\rangle$ and $|\phi\rangle$ are not orthogonal [7,14]. Consequently, one may want to have the best cloning machine, i.e., one that takes $|\Psi\rangle$ as close as possible to $|\psi\rangle|\psi\rangle$, and at the same time takes $|\Phi\rangle$ as close as possible to $|\phi\rangle|\phi\rangle$. The best cloning machine is one which maximizes the quantity $F_{clone} = (\langle\psi|\langle\psi||\Psi\rangle + \langle\phi|\langle\phi||\Phi\rangle)/2$ [11]. In the case of deleting, we want to have the *complementary* evolution $|\psi\rangle|\psi\rangle \rightarrow |\Psi_d\rangle$ and $|\phi\rangle|\phi\rangle \rightarrow |\Phi_d\rangle$ (in a closed system), where in the perfect case, we want to have $|\Psi_d\rangle = |\psi\rangle|0\rangle$ and $|\Phi_d\rangle = |\phi\rangle|0\rangle$, $|0\rangle$ being a fixed state from which information (whether it was $|\psi\rangle$ or $|\phi\rangle$) has been deleted. Again, this exact case is not possible under a quantum-mechanical operation, when $|\psi\rangle$ and $|\phi\rangle$ are nonorthogonal [8,12]. Therefore, just as in the case of cloning, one may again want to obtain $|\Psi_d\rangle$ as close as possible to $|\psi\rangle|0\rangle$, and at the same time $|\Phi_d\rangle$ as close as possible to $|\phi\rangle|0\rangle$. The best deleting machine is one that, for some fixed $|0\rangle$, maximizes the quantity $F_{delete} = (\langle\psi|\langle 0||\Psi_d\rangle + \langle\phi|\langle 0||\Phi_d\rangle)/2$ (cf. [12]).

We now show that

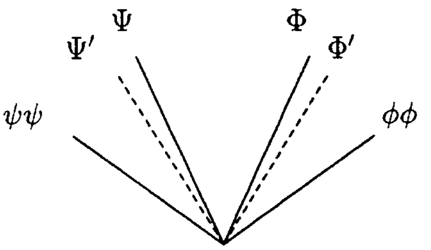

FIG. 1. A pictorial representation of the states $|\psi\rangle|\psi\rangle$, $|\phi\rangle|\phi\rangle$, $|\Psi\rangle$, $|\Phi\rangle$, $|\Psi'\rangle$, and $|\Phi'\rangle$.

Theorem 1: *In a physical system that follows (i), (ii), and (iii), for two nonorthogonal states ($|\psi\rangle$ and $|\phi\rangle$), cloning evolutions that allow fidelities that are better than the best quantum mechanically attainable fidelity F_{clone}, will result in signaling.*

Before proving the theorem, let us note that in [15] (cf. [16]), it was shown that a better fidelity than the best quantum-mechanical fidelity leads to signaling. In addition, Ref. [17] shows that exact deleting results in signaling. However, in both these cases, they considered *universal* cloning and deleting. Such cloning and deleting are invalidated by linearity. Here, however, we consider cloning and deleting of two nonorthogonal states, which cannot be ruled out by linearity. No cloning and no deleting of two nonorthogonal states can be proven by using unitarity, a more stricter restriction than just linearity. It has been widely regarded that violation of linearity will lead to signaling (cf. [18]). Our results show that important linear operations can also lead to signaling.

Proof: Let us consider symmetric cloning. However, all the considerations carry over, with a little more algebra, to the asymmetric case also. Suppose that for the input states $|\psi\rangle$ and $|\phi\rangle$, the best quantum mechanically attainable cloning fidelity is F_{clone}, and is attained with the states $|\Psi\rangle$ and $|\Phi\rangle$. Suppose also that there exists a (non-quantum) cloning machine that produces the states $|\Psi'\rangle$ and $|\Phi'\rangle$, giving a better fidelity $F'_{clone}=(\langle\psi|\langle\psi||\Psi'\rangle+\langle\phi|\langle\phi||\Phi'\rangle)/2$, that is $>F_{clone}$.

In Fig. 1, we give a pictorial representation of the states $|\psi\rangle|\psi\rangle$, $|\phi\rangle|\phi\rangle$, $|\Psi\rangle$, $|\Phi\rangle$, $|\Psi'\rangle$, and $|\Phi'\rangle$. Note that in general, e.g. $|\Psi\rangle$ and $|\Phi\rangle$ will not be in the same plane as $|\psi\rangle|\psi\rangle$ and $|\phi\rangle|\phi\rangle$. Consider the cone formed by $|\Psi\rangle$ and $|\Phi\rangle$. The angle (modulus of inner product) between these states must be the same as that between $|\psi\rangle|0\rangle$ and $|\phi\rangle|0\rangle$. This is due to the fact that unitary evolution preserves the inner product of evolved states. Thus, $|\psi\rangle|0\rangle$ and $|\phi\rangle|0\rangle$ must lie on the same cone as that of $|\Psi\rangle$ and $|\Phi\rangle$. Now, whenever $|\psi\rangle$ and $|\phi\rangle$ are nonorthogonal, we have $|\langle\psi|\langle\psi||\phi\rangle|\phi\rangle|<|\langle\psi|\langle 0||\phi\rangle|0\rangle|=|\langle\Psi||\Phi\rangle|$ This is why the cone of $|\psi\rangle|\psi\rangle$ and $|\phi\rangle|\phi\rangle$ is drawn to be wider than the cone of $|\Psi\rangle$ and $|\Phi\rangle$ in Fig. 1.

As $F'_{clone}>F_{clone}$, the cone formed by $|\Psi'\rangle$ and $|\Phi'\rangle$ will be wider than that formed by $|\Psi\rangle$ and $|\Phi\rangle$ (see Fig. 1). Since we consider symmetric cloning, all three cones will be coaxial. Thus, we have $|\langle\Psi'||\Phi'\rangle|<|\langle\Psi||\Phi\rangle|$. However, $|\langle\Psi||\Phi\rangle|=|\langle\psi|\langle 0||\phi\rangle|0\rangle|$, since $|\Psi\rangle$ and $|\Phi\rangle$ are produced from $|\psi\rangle|0\rangle$ and $|\phi\rangle|0\rangle$ by quantum-mechanical operations. Therefore, we have that $|\langle\Psi'||\Phi'\rangle|<|\langle\psi|\langle 0||\phi\rangle|0\rangle|$, a clear departure from quantum-mechanical evolutions (since the inner product must be preserved in quantum-mechanical evolutions). Whenever this relation holds, the von Neumann entropy of $\varrho_{out}=(|\Psi'\rangle\langle\Psi'|+|\Phi'\rangle\langle\Phi'|)/2$ is *greater* than the von Neumann entropy of $\varrho_{in}=(|\psi\rangle|0\rangle\langle\psi|\langle 0|+|\phi\rangle|0\rangle\langle\phi|\langle 0|)/2$.

Consider now the bipartite state

$$|\alpha\rangle=\frac{1}{\sqrt{2}}[|0\rangle_A(|\psi\rangle|0\rangle)_B+|1\rangle_A(|\phi\rangle|0\rangle)_B], \tag{1}$$

where $\langle 0||1\rangle=0$. Suppose that a super-quantum-mechanical cloning evolution, attaining F'_{clone} for the states $|\psi\rangle$ and $|\phi\rangle$, acts on part B of the state $|\alpha\rangle$, so that the state $|\alpha\rangle$ evolves into $|\alpha_1\rangle=(|0\rangle_A|\Psi'\rangle_B+|1\rangle_A|\Phi'\rangle_B)/\sqrt{2}$. Note that we have explicitly used linearity [item (iii)], in obtaining the state $|\alpha_1\rangle$. The local density matrices of the B part of the states $|\alpha\rangle$ and $|\alpha_1\rangle$ are ϱ_{in} and ϱ_{out}. We therefore have a difference in von Neumann entropy of the input and output states in the B part. Since $|\alpha\rangle$ and $|\alpha_1\rangle$ are pure states, this difference can be exactly verified in the A parts. Therefore, consequent upon action of any member of the family of super-quantum cloning evolutions (the family is generated by pairs of nonorthogonal states) in the B part, an increase in entropy can be observed in the A part. □

Similar reasoning holds for the case of deleting also. Only Fig. 1 must be replaced by one in which an outer cone is formed by $|\Psi_d\rangle$ and $|\Phi_d\rangle$ and an inner one formed by $|\psi\rangle|0\rangle$ and $|\phi\rangle|0\rangle$. The middle cone will again be formed by $|\Psi'_d\rangle$ and $|\Phi'_d\rangle$. Here $|\Psi_d\rangle$ and $|\Phi_d\rangle$ will represent the states that are obtained from $|\psi\rangle|\psi\rangle$ and $|\phi\rangle|\phi\rangle$, by the best quantum-mechanical deleting operation, assumed to be F_{delete}. In addition, the shared bipartite state that must be considered is $|\alpha'\rangle=(|0\rangle_A(|\psi\rangle|\psi\rangle)_B+|1\rangle_A(|\phi\rangle|\phi\rangle)_B)/\sqrt{2}$. In this case, a super-quantum deleting evolution in the B part, results in a decrease of entropy in the A part, so that

Theorem 2: *In a physical system that follows (i), (ii), and (iii), for two nonorthogonal states ($|\psi\rangle$ and $|\phi\rangle$), deleting evolutions that allow fidelities that are better than the best quantum mechanically attainable fidelity F_{delete}, will result in signaling.*

We will now show that it is possible to test the effect, by showing that the states $|\alpha\rangle$ and $|\alpha'\rangle$ (used in Theorems 1 and 2 above) can be prepared with current technology. Photons are as yet the best candidates for quantum communication. We give our strategy in terms of the polarization degree of freedom of photons.

The case of cloning. In this case, we require to prepare the state $|\alpha\rangle$ of Eq. (1). Let us write it as $(1/\sqrt{2})(|0\rangle_1|\psi\rangle_2+|1\rangle_1|\phi\rangle_2)|0\rangle_4$, where the photon 1 is to go to Alice (A) who is in the accessible part of the experiment. The photons 2 and 4 are to be sent to the environment, and will not be directly observed (see Fig. 2). For nonorthogonal $|\psi\rangle$ and $|\phi\rangle$, the first part $(|0\rangle|\psi\rangle+|1\rangle|\phi\rangle)/\sqrt{2}$ is a nonmaximally entangled state. It can, of course, be written in Schmidt decomposition as $a|0'\rangle|0''\rangle+b|1'\rangle|1''\rangle$, where a and b are positive numbers with $a^2+b^2=1$. We choose the local axes such that this nonmaximally entangled state is $a|V\rangle|H\rangle+b|H\rangle|V\rangle=|\beta\rangle$ (say), where $|V\rangle$ and $|H\rangle$ are, respectively, the vertical and horizontal po-

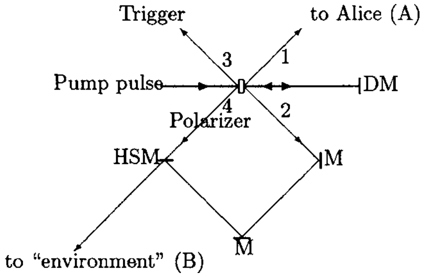

FIG. 2. Schematic description of the arrangement in the case of cloning. The down conversion crystal is denoted as a box, and delay mirror, mirrors, and half-silvered mirror are denoted, respectively, by DM, M, and HSM. See text for details.

larizations of a photon. This can be prepared by spontaneous pulsed parametric downconversion [19,20].

A schematic description of the arrangement is given in Fig. 2. A pump laser is directed towards a downconversion crystal. There is then a certain probability of obtaining the state $|\psi^{+}\rangle=(|V\rangle_1|H\rangle_2+|H\rangle_1|V\rangle_2)/\sqrt{2}$ in the modes 1 and 2 [21]. Subsequently, local fitering operations are performed to create the nonmaximally entangled state $|\beta\rangle=a|V\rangle_1|H\rangle_2+b|H\rangle_1|V\rangle_2$ in modes 1 and 2. (These local filtering operations are not shown in the figure.) After passing through the crystal, the pulse is reflected back to the crystal by a delay mirror (see, e.g., [22]). There is again a certain probability of creation of a pair in the state $|\psi^{+}\rangle$ in modes 3 and 4. We consider only those cases when *both* the pairs are created. Mode 3 is detected and acts as a trigger to indicate that a photon is actually present in mode 4. The polarization of the photon in mode 4 is set to vertical by using a polarizer. Thus, the photon in mode 4 is ultimately in the state $|V\rangle$, and this acts as our blank state $|0\rangle_4$ in the total state $|\alpha\rangle_{124}=|\beta\rangle_{12}|0\rangle_4$. Mode 4 and mode 2 (after being reflected by two mirrors) are directed to a half-silvered mirror, so that mode 4 passes through and mode 2 is reflected. The delay in the creation of pair 34 is made such that the photons in modes 2 and 4 reach the half-silvered mirror at the same time. Then these two photons are directed to the environment. The photon in mode 2 runs towards Alice (A), and remains in the accessible part of the experiment.

Here we are using Type II downconversion [23]. In the Type I case, the path degrees of freedom are used for entanglement generation. This is a problem here, as we want the B part photons to ultimately be directed towards a single direction. Note here that we have *not* used entanglement swapping [24,25] to prepare our entangled state. Here, photon 3 acts as a trigger for guaranteeing the existence of photon 4, while the photon 1 will subsequently be detected by Alice (and will act as a trigger for the state created in modes 12), and we consider only those runs of the experiment, in which both the trigger photon 3 and photon 1 are detected.

The case of deleting. In this case, we must prepare the state $|\alpha'\rangle$. This can be obtained after local filtering operations on a Greenberger-Horne-Zeilinger (GHZ) state [26] $[|0\rangle_A(|0\rangle|0\rangle)_B+|1\rangle_A(|1\rangle|1\rangle)_B]/\sqrt{2}$, after which the first part remains in the accessible part (A) of the experiment and the second and third parts are aligned to a single direction (just as in Fig. 2 in case of cloning) and sent to the environment. Experimental observation of the GHZ state has been reported in [27]. However, the experiment relies for its success on actual observation of *all* the photons that make up the GHZ state (plus a trigger photon). Whereas this is sufficient for many important purposes, it is not sufficient for us. In our case, at least two photons are not to be directly observed. However, in a proposal for preparation for the GHZ state [28], the state is prepared without the restriction of having to actually detect the photons (making up the GHZ), to know that a GHZ state is produced. After production of a GHZ by this proposal, local filtering operations can be carried out to produce the state $|\alpha'\rangle$.

After the photons in the B part are sent to the environment, Alice makes measurements on her photon to determine the von Neumann entropy of her state. The von Neumann entropy can conveniently be found by measurement results from outcomes in a Mach-Zehnder interferometer, to which the photon in mode 1 can be directed into. More economical methods, although requiring measurements over many copies, can be found in Refs. [29]. The von Neumann entropy of the A part of the state $|\alpha\rangle$, or the 1 part of the state $|\beta\rangle_{12}|V\rangle_4$ is $H(a^2)=-a^2\log_2 a^2-b^2\log_2 b^2$ Similarly, let the von Neumann entropy of the A part of the state $|\alpha'\rangle$ be $H(a'^2)$. As we have seen in Theorem 1 above, any departure from the value $H(a^2)$ in the experiment for cloning, or from the value $H(a'^2)$ in the experiment for deleting, of the von Neumann entropy of the polarization degrees of freedom of the photon 1, as detected by Alice from her experimental results, will indicate a signaling. This in turn indicates that there are non-quantum-mechanical operations that have acted on the modes 2 and 4, that were directed to the environment.

The same experiment can be carried on for different values of a, a'. The values of a, a' can be varied by varying the parameters of the local filtering apparatus. Each set of $\{a,a'\}$, checks for a duo of non-quantum-mechanical evolutions, one from super-quantum-mechanical cloning, and the other from super-quantum-mechanical deleting. Thus, we can check for two *complementary* families of possible non-quantum-mechanical evolutions on the modes 2 and 4. In an actual experiment, there will be some noise. The results obtained from such experiments can be used to put bounds on the power of possible non-quantum-mechanical evolutions in the environment.

In principle, the "environment" can be some extreme situations, e.g., an evaporating black hole, where conditions may be far too extreme for the laws established in the usual laboratories to be applicable (see, e.g., [30], cf. [31]). However, just as in the recent proposal [30], the way to send the probes (the photons 2 and 4 in our case) to an evaporating black hole, remains a problem. However, let us mention that we consider a bipartite state instead of the three-party state of [30]. Another conceivable situation is where a person claims to be able to perform non-quantum operations, but denies direct access to his/her laboratory. Our procedure can then be used to check for his/her claim. However the main impetus, in this paper (of the Theorems 1 and 2), is to have an independent reason for believing in the quantum dynamics.

In conclusion, we have shown that in a physical system that follows (i), (ii), and (iii), a cloning operation acting in a region B, that leads to a better than quantum-mechanical fidelity, results in signaling to a far-apart region A. The same conclusion can be obtained for deleting. The strategy to check for such signaling does not require to perform measurements in the region B. The two-party states required to perform the strategy can be prepared with current technology. This gives us an independent basis to believe in the quantum dynamics.

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