Surface Gap Solitons

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We put forward the existence of *surface gap solitons* at the interface between uniform media and an optical lattice with defocusing nonlinearity. Such new type of solitons forms when the incident and reflected waves at the interface of the lattice experience Bragg scattering, and feature a combination of the unique properties of both surface waves and gap solitons. We discover that gap surface solitons exist only when the lattice depth exceeds a threshold value, that they can be made completely stable, and that they can form stable bound states.

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Surface waves are a special type of waves, which are confined at the very boundary between two different media. They feature unique properties as well as potential for applications, e.g., for exploration of intrinsic and extrinsic surface characteristics. Special attention has been devoted to the nonlinear surface waves in solid-state physics, nonlinear optics, and near-field optics [1–4]. Interaction of waves with nonlinear interfaces gives rise to a number of important phenomena such as optical bistability and hopping between regimes of total internal reflection and complete transmission [5,6]. The properties of waves supported by nonlinear interfaces are now well established, and nonlinear surface waves were observed in photorefractive crystals with specific anisotropic diffusion nonlinearity [7]. However, by and large, experimental progress has been severely limited by the giant refractive-index changes that are required for nonlinear surface waves to self-sustain themselves in natural materials [2], hence most theoretical predictions are still awaiting experimental confirmation. Thus, elucidation of settings where surface waves exist under realizable conditions is of paramount importance.

One potential avenue put forward recently in Refs. [8,9] is the interface between uniform and periodic layered media, including photonic crystals and metamaterials, which are known to support linear surface waves [10], including Dyakonov waves [11]. The periodic refractive-index modulation modifies the properties of nonlinear excitations as well. Suitable periodic structures with engineered and thus tunable properties can be made in waveguide arrays featuring small enough refractive-index changes in each waveguide with technology currently available. Optical induction of refractive-index landscapes [12–14] might offer a potential powerful alternative.

Optical lattices support different types of solitons, including ground state, twisted solitons, complex soliton trains, or vortex solitons [12–16]. So-called *gap solitons* are also possible. They form by the nonlinear coupling between forward- and backward-propagating waves when both experience Bragg scattering from periodic structure [17]. Gap solitons exist in photonic crystals and layered structures [18,19], fiber Bragg gratings [20], Bose-Einstein condensates [21], and waveguide arrays [22,23]. Incoherently coupled solitons can form vector complexes [24–26]. The stability of optical gap solitons in Bragg gratings was studied in Ref. [27]. Gap solitons in bulk optical lattices were studied theoretically [28] and observed experimentally [12,29].

In this Letter we put forward the new concept of a *surface gap soliton*, an entity we found to exist at the interface of uniform media and optical lattice with defocusing nonlinearity. Such a new type of solitons combines specific features typical for nonlinear surface waves and the unique properties exhibited by gap solitons. Despite the defocusing character of the nonlinearity of both uniform and periodic media forming the interface, gap surface solitons are spatially localized at the interface. As mentioned above, the possibility of generation of lattices with engineered interface properties (e.g., the lattice depth and period) might afford a drastic reduction of the light intensity required to generate the self-sustained surface waves, opening the door to the experimental observation of our predictions.

We consider beam propagation at the interface of uniform and periodic media with Kerr-type cubic defocusing nonlinearity described by the nonlinear Schrödinger equation for dimensionless amplitude of the light field q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} + q|q|^2 - pR(\eta)q. \qquad (1)$$

Here the transverse η and longitudinal ξ coordinates are scaled in terms of beam width and diffraction length, respectively. The parameter p characterizes the depth of refractive-index modulation. The function $R(\eta)$ stands for the lattice refractive-index profile and is given by $R(\eta) = 1 - \cos(\Omega\eta)$ at $\eta \geq 0$ and $R(\eta) = 0$ at $\eta < 0$, where Ω is the modulation frequency. Notice that since the function $R(\eta)$ describing the lattice profile in Eq. (1) oscillates between 0 and 2 in contrast to function $\cos^2(\Omega\eta/2)$ typically used for lattice profiles in the literature [12–16], parameter p amounts to twice the lattice depth as it defined in [12–16]. We assume that the depth of the linear refractive-index modulation is small compared with the unperturbed index and is of the order of nonlinear correction due to the Kerr effect. Such refractive-index landscapes can be fabricated, e.g., by etching waveguide arrays on top of a substrate [22,23], or they might be induced optically by an interference pattern in a photorefractive crystal using vectorial interactions [12–14]. A sharp transition between the uniform region and the lattice might be created by erasing part of the red-light imprinted lattice with an intense green-light wave creating a region with high photoconductivity. This is feasible, for example, in $Sr_{1-x}Ba_xNb_2O_6$ crystals whose photosensitivity strongly depends on wavelength. Notice that Eq. (1) admits several conserved quantities, including the energy flow U and the Hamiltonian H:

$$U = \int_{-\infty}^{\infty} |q|^2 d\eta, \qquad H = \frac{1}{2}\int_{-\infty}^{\infty} [|\partial q/\partial\eta|^2 - 2pR(\eta)|q|^2 + |q|^4] d\eta. \tag{2}$$

To understand the main properties of gap surface solitons it is important to consider first the Floquet-Bloch spectrum of linear periodic structure. In doing so we search for solutions of the linear version of Eq. (1) with $R(\eta) = 1 - \cos(\Omega\eta)$, where $\eta \in (-\infty, +\infty)$, in the form $q(\eta, \xi) = w(\eta)\exp(ib\xi + ik\eta)$, where b is real propagation constant, k is the Bloch wave number, $w(\eta) = w(\eta + 2\pi/\Omega)$ is the complex periodic function. Substitution of the light field in such form yields the eigenvalue problem

$$bw = \frac{1}{2}\left(\frac{d^2w}{d\eta^2} + 2ik\frac{dw}{d\eta} - k^2 w\right) + pRw \tag{3}$$

that we solved numerically. The Floquet-Bloch spectrum $b(p)$ of the (infinite) lattice is shown in Fig. 1(a) (here and throughout the Letter we let $\Omega = 4$). Inside the transmission bands (gray regions) Eq. (1) admits periodic Bloch wave solutions, while in the gaps (white regions) periodic waves do not exist. The Floquet-Bloch spectrum possesses single semi-infinite gap and infinite number of finite gaps. When nonlinearity of the medium is taken into account, solitons appear as defect modes that are located inside the gaps of the Floquet-Bloch spectrum. In the case of defocusing nonlinearity the simplest gap solitons can be found in the first finite gap [12,28,29].

We now consider the interface between the lattice and a uniform medium. The presence of the interface drastically modifies the properties of the gap solitons. We searched for their profiles numerically in the form $q(\eta, \xi) = w(\eta) \times \exp(ib\xi)$, where $w(\eta)$ is a real function. Profiles of the simplest gap solitons associated with the first finite gap of Floquet-Bloch spectrum are shown in Figs. 2(a)–2(d). Surface gap solitons are spatially localized despite the defocusing character of the nonlinearity of both uniform and periodic media. Thus, the periodic refractive-index modulation may result in existence of localized surface waves at the defocusing interfaces, in clear contrast to the case of single interface between defocusing media [2]. Moreover, the defocusing nonlinearity results in suppression of modulational instability along second uniform transverse coordinate ζ that was not included into Eq. (1).

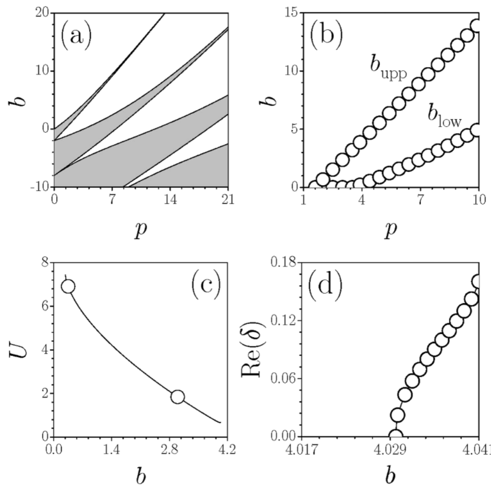

FIG. 1. (a) Band-gap structure of periodic lattice. Bands are marked with gray color; gaps are shown white. (b) Lower and upper cutoffs for existence of surface solitons originating from the first finite gap. (c) Energy flow vs propagation constant at $p = 4$. Points marked by circles in (c) correspond to solitons depicted in Figs. 2(c) and 2(d). (d) Real part of perturbation growth rate vs propagation constant at $p = 4$. Notice the huge difference of scale between horizontal axes in (c) and (d).

Physically, gap surface solitons are formed due to the nonlinear coupling between forward and backward-propagating waves when both of them experience Bragg scattering in the lattice. The presence of exponentially decaying tails in the uniform medium implies that gap solitons may exist only for $b > 0$. This constraint together with the structure of Floquet-Bloch spectrum defines the domain of existence of surface gap solitons. One may expect to find solitons associated with the first finite gap only when the lattice depth exceeds a critical value $p_{cr}^{(1)} \approx 1.32$ at which the upper edge of the first gap reaches $b = 0$ [Fig. 1(a)]. Actually, the critical value of the lattice depth is a bit higher ($p_{cr}^{(1)} \approx 1.61$) because there is an upper cutoff b_{upp} for existence of solitons that is smaller than the upper gap edge [see Fig. 1(c)]. Notice that U decreases with b, except for the very narrow region near upper cutoff, not even visible in Fig. 1(c), where $dU/db > 0$.

In sharp contrast to bulk solitons, surface gap solitons do not occupy the whole band gap. Physically, this occurs because of the conditions imposed by the interface. Gap solitons supported by infinite lattices broaden drastically near upper gap edge, while their peak amplitude monotonically decreases. In contrast, the field penetration depth into uniform medium for a soliton residing at interface is dictated by the value of its propagation constant. When b approaches the upper gap edge, solitons supported by the interface expand substantially into the lattice, but they cannot expand into the uniform medium since already for moderate lattice depths $p \sim 2$ one has $b > 1$ near upper gap edge. This competition results in a *decrease* of the

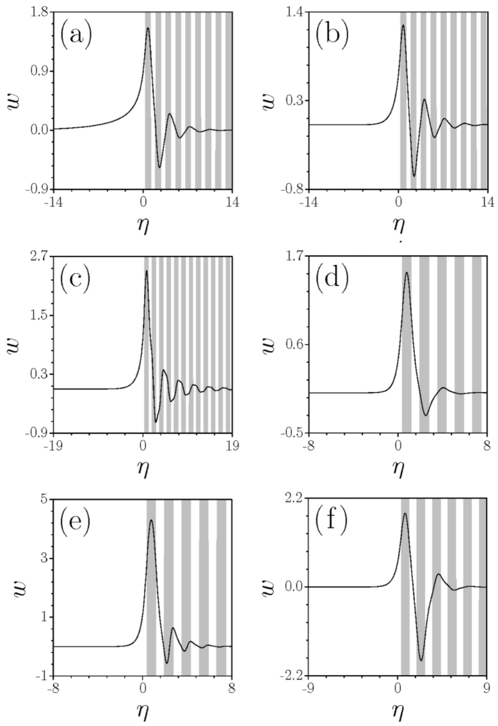

FIG. 2. Gap surface solitons originating from the first finite gap at (a) $b = 0.02$, $p = 2$, (b) $b = 0.6$, $p = 2$, (c) $b = 0.35$, $p = 4$, (d) $b = 3$, $p = 4$. (e) Soliton originating from the second finite gap at $b = 3$, $p = 12$. (f) Twisted soliton at $b = 2$, $p = 4$. Gray regions correspond to $R(\eta) > 0$; white regions correspond to $R(\eta) \leq 0$.

soliton width as $b \to b_{upp}$ and appearance of cutoff b_{upp} for soliton existence. With increase of the lattice depth b_{upp} gradually approaches the upper edge of the first gap. The lower cutoff b_{low} for existence of gap surface solitons remains zero until p reaches the second critical value $p_{cr}^{(2)} \approx 3.56$ at which the lower edge of first gap reaches $b = 0$. For $p \geq p_{cr}^{(2)}$, the lower cutoff coincides with the lower gap edge [Fig. 1(b)]. The behavior of the gap solitons near the lower cutoff depends substantially on the lattice depth. Thus, for $p_{cr}^{(1)} \leq p \leq p_{cr}^{(2)}$ solitons near lower cutoff feature slowly decaying tails in the uniform medium [Fig. 2(a)]. In contrast, for $p > p_{cr}^{(2)}$ solitons that may be well localized close to the upper cutoff [Fig. 2(d)] feature long oscillating tails in the lattice region when $b \to b_{low}$ [Fig. 2(c)]. Consequently, the integral soliton width $W = U^{-1} \int_{-\infty}^{\infty} \eta |q|^2 d\eta$ always diverges in the lower cutoff.

To elucidate the stability of surface gap solitons, we searched for perturbed solutions of Eq. (1) in the form $q(\eta, \xi) = [w(\eta) + u(\eta, \xi) + iv(\eta, \xi)] \exp(ib\xi)$, where u, v are real and imaginary parts of perturbation that can grow with complex rate δ upon propagation. Linearization of Eq. (1) around w yields the eigenvalue problem

$$\delta u = -\frac{1}{2}\frac{d^2 v}{d\eta^2} + bv - pRv + w^2 v,$$
$$\delta v = \frac{1}{2}\frac{d^2 u}{d\eta^2} - bu + pRu - 3w^2 u, \tag{4}$$

which we solved numerically to find perturbation profiles and associated growth rates δ. A comprehensive stability analysis showed that surface gap solitons in the first gap exhibit exponential instability in an extremely narrow region near the upper cutoff, where $dU/db > 0$ [see Fig. 1(d) for dependence of growth rate δ on the propagation constant]. For $b < b_{cr}$ (where b_{cr} corresponds to onset of exponential instability) gap solitons are *completely stable* [Fig. 1(d)]; this *stability domain* occupies *almost all* the total soliton existence domain. Linear stability analysis also revealed the presence of very weak oscillatory instability with $\mathrm{Re}\,\delta \ll \mathrm{Im}\,\delta$ in a narrow region in the vicinity of lower cutoff. Instability of this type occurs because of resonant energy redistribution between the gaps and is specific to gap solitons [see also Refs. [21,27,28] where similar behavior was encountered for gap solitons in infinite periodic systems]. Perturbation eigenmodes associated with these weak oscillatory instabilities are poorly localized. Thus, the conclusion of the stability analysis is that except in narrow regions close to cutoffs of soliton existence almost all surface gap solitons of physical interest are stable. To test the predictions of linear stability analysis we solved Eq. (1) numerically with the input conditions $q(\eta, \xi = 0) = w(\eta)[1 + \rho(\eta)]$, where $\rho(\eta)$ is the random function with Gaussian distribution and variance σ_{noise}^2. Figures 3(a) and 3(b) illustrate the stable propagation of the perturbed surface solitons associated with the first gap. Such gap solitons retain their input structure during the distances that exceed the experimentally feasible crystal lengths by several orders of magnitude.

To address the excitation of the solitons we performed series of simulations with input conditions that departure substantially from exact soliton profiles. We found that

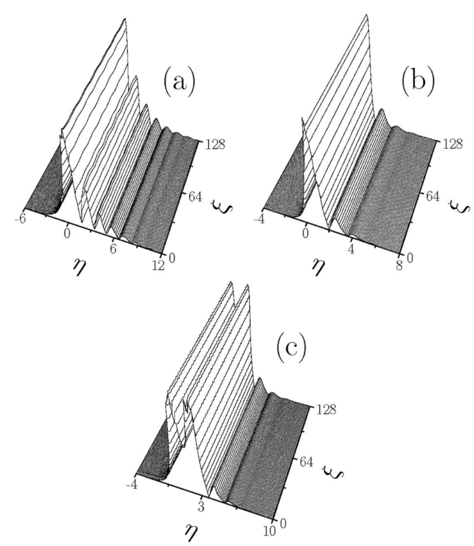

FIG. 3. Stable propagation of simplest surface gap solitons at (a) $b = 0.6$, $p = 2$, and (b) $b = 3$, $p = 4$, and stable propagation of even surface gap soliton at (c) $b = 2$, $p = 4$. The white noise with variance $\sigma_{noise}^2 = 0.01$ was added into stationary profiles at $\xi = 0$.

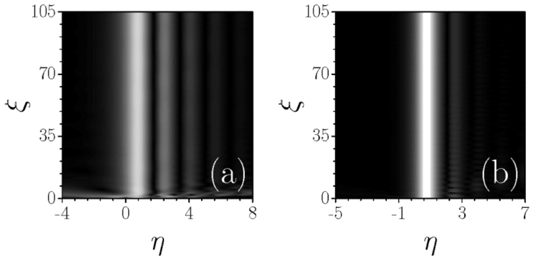

FIG. 4. Excitation of gap surface solitons with (a) a pair of interfering Gaussian beams at $p = 2$ and (b) a single Gaussian beam at $p = 4$.

surface gap solitons can be excited from interference pattern produced by two counterpropagating Gaussian beams $A\exp\{-[(\eta-\eta_0)/w_0]^2 \pm i\alpha(\eta-\eta_0)\}$. Figure 4(a) shows excitation dynamics for $A = 0.825$, $\eta_0 = \pi/\Omega$, $\alpha = \Omega/2$, and $w_0 = 2.25$. After a fast initial reshaping and radiation emission a steadily propagating gap soliton emerges. In deeper lattices gap surface solitons can be excited even with a single Gaussian beam $A\exp\{-[(\eta-\eta_0)/w_0]^2\}$, as illustrated in Fig. 4(b).

Besides surface solitons associated with the first gap, we have found a number of stable solitons that are associated with other finite gaps. An example of soliton profile originating from the second gap is shown in Fig. 2(e). The main properties of such solitons are similar to those featured by solitons originating from the first gap, except for the fact that critical values of the lattice depths $p_{\rm cr}^{(1)}$, $p_{\rm cr}^{(2)}$ defining the existence domain for second-gap solitons are substantially higher. Several surface gap solitons can form bound states, similarly to gap solitons in infinite lattices [15]. Propagation of a stable "even" surface gap soliton, that can be viewed as a combination of two in-phase solitons, is shown in Fig. 3(c). The properties of even solitons and domain of their existence are close to that for simplest surface solitons. In contrast, out-of-phase soliton combinations, resulting in formation of "twisted" soliton [Fig. 2(f)], are exponentially unstable in the entire domain of their existence, while Re δ monotonically increases as $b \to b_{\rm low}$.

The results described above imply that nonlinearity introduces unique features into the properties of surface waves, as opposed to the case of a linear interface between uniform and periodic layered media [10]. This includes enhanced robustness, as well as the possibility of formation of nonlinear bound states of several solitons.

Summarizing, we introduced surface gap solitons supported by the interface between uniform media and optical lattices with defocusing Kerr-type nonlinearity. Such solitons combine in a nontrivial fashion the unique features that are typical for surface waves supported by nonlinear interfaces, and those exhibited by gap solitons existing in periodic media. Fabrication of waveguide arrays featuring small refractive-index variations, as well as induction of reconfigurable optical lattices, in suitable nonlinear materials should make possible the observation of the new type of surface solitons introduced here, with experimentally feasible light intensities.

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[1] *Nonlinear Waves in Solid State Physics*, edited by A. D. Boardman, M. Bertolotti, and T. Twardowski, NATO ASI Vol. 247 (Plenum, New York 1989); *Nonlinear Surface Electromagnetic Phenomena*, edited by H. E. Ponath and G. I. Stegeman (North-Holland, Amsterdam, 1991).
[2] G. I. Stegeman *et al.*, J. Appl. Phys. **58**, R57 (1985); J. Lightwave Technol. **6**, 953 (1988).
[3] D. Mihalache, M. Bertolotti, and C. Sibilia, Prog. Opt. **27**, 229 (1989).
[4] *Near-Field Optics and Surface Plasmon Polaritons*, edited by S. Kawata (Springer, New York, 2001).
[5] A. E. Kaplan, JETP Lett. **24**, 114 (1976); Sov. Phys. JETP **45**, 896 (1977).
[6] P. W. Smith *et al.*, Appl. Phys. Lett. **35**, 846 (1979).
[7] M. Cronin-Golomb, Opt. Lett. **20**, 2075 (1995).
[8] K. G. Makris *et al.*, Opt. Lett. **30**, 2466 (2005).
[9] J. Hudock *et al.*, Opt. Express **13**, 7720 (2005).
[10] P. Yeh, A. Yariv, and A. Y. Cho, Appl. Phys. Lett. **32**, 104 (1978).
[11] D. Artigas and L. Torner, Phys. Rev. Lett. **94**, 013901 (2005).
[12] J. W. Fleischer *et al.*, Phys. Rev. Lett. **90**, 023902 (2003); Nature (London) **422**, 147 (2003).
[13] D. Neshev *et al.*, Opt. Lett. **28**, 710 (2003).
[14] H. Martin *et al.*, Phys. Rev. Lett. **92**, 123902 (2004).
[15] Y. V. Kartashov *et al.*, Opt. Express **12**, 2831 (2004).
[16] J. Yang and Z. Musslimani, Opt. Lett. **28**, 2094 (2003).
[17] W. Chen and D. L. Mills, Phys. Rev. Lett. **58**, 160 (1987).
[18] N. Akozbek and S. John, Phys. Rev. E **57**, 2287 (1998).
[19] S. F. Mingaleev and Y. S. Kivshar, Phys. Rev. Lett. **86**, 5474 (2001).
[20] B. J. Eggleton *et al.*, Phys. Rev. Lett. **76**, 1627 (1996).
[21] B. Eiermann *et al.*, Phys. Rev. Lett. **92**, 230401 (2004); O. Zobay *et al.*, Phys. Rev. A **59**, 643 (1999); P. J. Y. Louis *et al.*, Phys. Rev. A **67**, 013602 (2003); N. K. Efremidis and D. N. Christodoulides, Phys. Rev. A **67**, 063608 (2003).
[22] D. Mandelik *et al.*, Phys. Rev. Lett. **90**, 053902 (2003).
[23] D. Mandelik *et al.*, Phys. Rev. Lett. **92**, 093904 (2004).
[24] O. Cohen *et al.*, Phys. Rev. Lett. **91**, 113901 (2003).
[25] A. A. Sukhorukov and Y. S. Kivshar, Phys. Rev. Lett. **91**, 113902 (2003).
[26] Y. V. Kartashov *et al.*, Phys. Rev. E **70**, 066623 (2004).
[27] I. V. Barashenkov, D. E. Pelinovsky, and E. V. Zemlyanaya, Phys. Rev. Lett. **80**, 5117 (1998).
[28] D. E. Pelinovsky, A. A. Sukhorukov, and Y. S. Kivshar, Phys. Rev. E **70**, 036618 (2004).
[29] D. Neshev *et al.*, Phys. Rev. Lett. **93**, 083905 (2004).