Multipole vector solitons in nonlocal nonlinear media

Yaroslav V. Kartashov and Lluis Torner

ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, and Universitat Politecnica de Catalunya, 08860 Castelldefels (Barcelona), Spain

Victor A. Vysloukh

Departamento de Fisica y Matematicas, Universidad de las Americas–Puebla, Santa Catarina Martir, 72820, Puebla, Mexico

Dumitru Mihalache

National Institute of Physics and Nuclear Engineering, P.O. Box MG-6, Bucharest, Romania

Received January 17, 2006; revised February 28, 2006; accepted March 2, 2006; posted March 9, 2006 (Doc. ID 67297)

We show that multipole solitons can be made stable via vectorial coupling in bulk nonlocal nonlinear media. Such vector solitons are composed of mutually incoherent nodeless and multipole components jointly inducing a nonlinear refractive index profile. We found that stabilization of the otherwise highly unstable multipoles occurs below certain maximum energy flow. Such a threshold is determined by the nonlocality degree.
© 2006 Optical Society of America
OCIS codes: 190.5530, 190.4360, 060.1810.

Nonlocality of the nonlinearity is a property exhibited by many nonlinear optical materials. For example, nonlocality may be important in nematic liquid crystals,[1,2] thermal self-action,[3] plasmas,[4] or photorefractive materials.[5] Nonlocality suppresses modulational instability,[6,7] and it stabilizes vortex[8] and 2D fundamental solitons.[9] Since interactions of solitons in nonlocal media are determined by spatial separation, out-of-phase beams could form bound states.[10,11] The maximum number of solitons that can be packed into stable bound state depends on the nature of nonlocal response.[12] Bound states of dark solitons were addressed in Refs. 13 and 14.

Two-dimensional bright solitons also form bound states in nonlocal media.[5,15] Recently, stable dipole solitons in a medium with a Gaussian response function have been predicted.[16] However, many actual materials exhibit nonlocal responses with profiles that depart drastically from the Gaussian one. In this Letter we address multipole solitons in a model with a nonlocal response of the Helmholtz type, encountered, in particular, in nematic liquid crystals and plasmas, and show that the shape of nonlocal response is *crucial* for the stability of 2D bound states. Thus *all* scalar bound states are found to be *unstable*, but they can be stabilized via vectorial coupling with nodeless solitons. Notice that multipole vector solitons were also studied in the local saturable or photorefractive media.[17–25]

We consider propagation of two mutually incoherent laser beams along the ξ axis in media with a nonlocal focusing nonlinearity described by the system of equations for light field amplitudes $q_{1,2}$ and nonlinear contribution to refractive index n,

$$i\frac{\partial q_{1,2}}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \zeta^2}\right) q_{1,2} - q_{1,2} n,$$

$$n - d\left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \zeta^2}\right) n = |q_1|^2 + |q_2|^2, \qquad (1)$$

where η, ζ, and ξ stand for normalized transverse and longitudinal coordinates scaled to the beam width and diffraction length, respectively. The parameter $d>0$ stands for the nonlocality degree of the nonlinear response. When $d\to 0$, Eqs. (1) reduce to Manakov vector nonlinear Schrödinger equations; the case $d\to\infty$ corresponds to the strongly nonlocal regime. Under proper conditions, Eqs. (1) describe nonlocal nonlinearities of partially ionized plasmas resulting from many-body interactions[4] and orientational nonlinearity of nematic liquid crystals (see Refs. 1 and 2 for details of derivation). Equations (1) conserve the energy flow $U=U_1+U_2=\iint_{-\infty}^{+\infty}(|q_1|^2+|q_2|^2)\mathrm{d}\eta \mathrm{d}\zeta$. The nonlinear contribution to the refractive index is given by $n(\eta,\zeta)=\iint_{-\infty}^{+\infty}G(\eta-\lambda,\zeta-\tau)[|q_1(\lambda,\tau)|^2+|q_2(\lambda,\tau)|^2] \times \mathrm{d}\lambda \mathrm{d}\tau$ where the response function $G(\eta,\zeta) =(2\pi d)^{-1}K_0[d^{-1/2}(\eta^2+\zeta^2)^{1/2}]$ is expressed in terms of zero-order MacDonald function. Notice that in contrast to the Gaussian response function of Ref. 16, $G(\eta,\lambda)$ has a logarithmic singularity at $\eta^2+\zeta^2\to 0$ and decays slowly. From now on we will refer to it as the Helmholtz response function.

We searched for soliton solutions of Eqs. (1) numerically in the form $q_{1,2}(\eta,\zeta,\xi)=w_{1,2}(\eta,\zeta) \times\exp(ib_{1,2}\xi)$, where $w_{1,2}$ are real functions, and $b_{1,2}$ are propagation constants. The standard relaxation method was employed that allows us to obtain soliton profiles with high accuracy (the difference between calculated profiles for subsequent iterations can be made less than 10^{-16}). In the scalar case ($w_1\equiv 0$, $w_2 \neq 0$) we found a variety of solutions composed of several (or single) bright spots with opposite phases that are arranged in rings. Such solitons exist in nonlocal media because the refractive index change in the

 © 2006 Optical Society of America

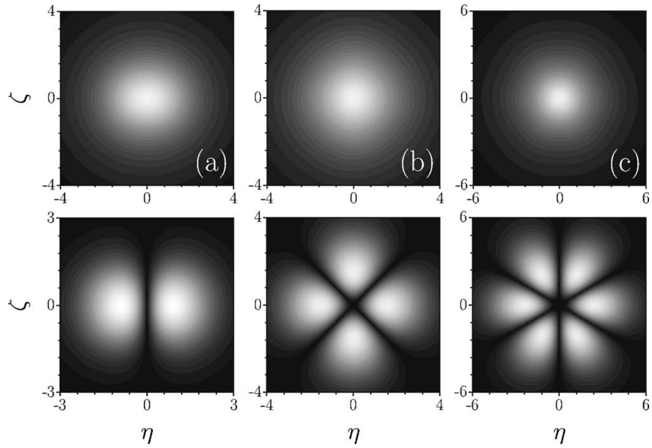

Fig. 1. Total refractive index profile (top row) and field modulus distribution in the second component (bottom) for (a) dipole soliton at $b_1=3$, $b_2=1.8$, and $d=3$, (b) quadrupole soliton at $b_1=3$, $b_2=0.9$, and $d=4$, (c) hexapole soliton at $b_1=3$, $b_2=0.4$, and $d=5$.

overlap region between neighboring spots is determined by the total intensity distribution in the entire transverse plane and under proper conditions equilibrium configurations of out-of-phase beams are possible. However, we found that all 2D scalar multipole solitons corresponding to the Helmholtz response are unstable and only nodeless solitons can be stable, in contrast to findings reported in Ref. 16 for Gaussian nonlocal responses.

Thus a first central result of this Letter is that the physical nature of the nonlocal response is crucial for stability of higher-order solutions in bulk geometries. Second, we found that vectorial coupling may lead to stabilization of vector multipole solitons even for realistic Helmholtz nonlocal response. Such solitons with $w_1 w_2 \neq 0$ were found at $b_2 \leqslant b_1$ (further, we set $b_1=3$, and vary b_2 and d). Profiles of the simplest vector solitons are shown in Fig. 1. We do not depict the first nodeless component w_1, but instead show the refractive index n and modulus of multipole component w_2. With the growth of nonlocality degree d, the width of the refractive index distribution remarkably increases, so that for $d \gg 1$ it greatly exceeds the width of intensity distribution. The energy flow increases with b_2 [Fig. 2(a)]. At fixed b_1 and d there exist lower b_2^{low} and upper b_2^{upp} cutoffs, so that vector solitons can be found only for $b_2^{\text{low}} \leqslant b_2 \leqslant b_2^{\text{upp}}$. When $d \to 0$, one has $b_2^{\text{upp}} \to b_1$. The width of existence domain shrinks with the increase of d [Figs. 2(b) and 2(e)] In the upper cutoff, w_1 vanishes, and vector solitons transform into scalar multipoles; in the lower cutoff, w_2 vanishes, and one gets scalar nodeless soliton. This is illustrated in Fig. 2(d) showing energy sharing $S_{1,2}=U_{1,2}/U$ between dipole soliton components versus b_2.

In the quasi-local medium ($d \ll 1$) and at $b_2 \to b_2^{\text{upp}}$, multipole vector solitons transform into several well-separated monopole vector solitons (the number of solitons is equal to the number of bright spots in the w_2 field) with weak in-phase w_1 and strong out-of-phase w_2 components that are both nodeless. In this case, the refractive index distribution features several well-separated peaks. When $b_2 \to b_2^{\text{low}}$, the strong w_1 component remains localized, while the weak w_2 component remarkably broadens. In the strongly nonlocal medium ($d \gg 1$), both w_1 and w_2 components are well localized in cutoffs. The refractive index distribution is bell shaped at $b_2 \to b_2^{\text{low}}$, but at $b_2 \to b_2^{\text{upp}}$, small peaks whose positions coincide with the positions of intensity maxima in the w_2 component are observable in otherwise smooth and wide refractive index profiles. The nodeless component also features a smooth bell-like shape at $b_2 \to b_2^{\text{low}}$ with a maximum at $\eta,\zeta=0$, while as $b_2 \to b_2^{\text{upp}}$, secondary peaks gradually appear in the positions corresponding to the intensity maxima in the w_2 component. The presence of peaks in the refractive index, even in the strongly nonlocal regime, is a specific feature of the Helmholtz response in comparison with the Gaussian response, where the refractive index profile created by multipole beam can be bell shaped in the strongly nonlocal case.

The transformation of a vector soliton into a stable scalar nodeless beam at $b_2 \to b_2^{\text{low}}$, and an unstable multipole beam at $b_2 \to b_2^{\text{upp}}$ suggests the existence of the stability domain for composite vector states near b_2^{low}. They become stable when the energy flow car-

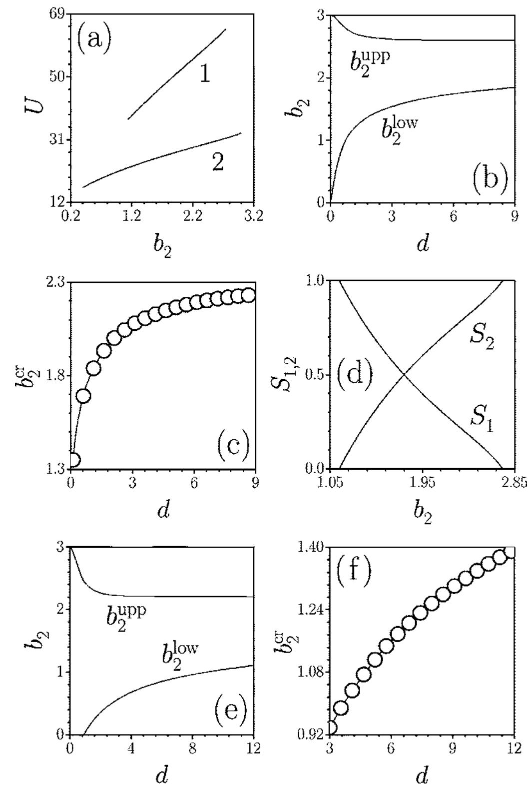

Fig. 2. (a) Energy flow of dipole soliton versus propagation constant b_2 at $d=1$ (line 1) and 0.2 (line 2). (b) Domain of existence of dipole solitons on the plane (d, b_2). (c) Critical propagation constant versus nonlocality degree for dipole solitons. (d) Energy sharing between components of dipole soliton versus propagation constant b_2. (e) Domain of existence of quadrupole solitons on the plane (d, b_2). (f) Critical propagation constant versus nonlocality degree for quadrupole solitons. In all cases $b_1=3$.

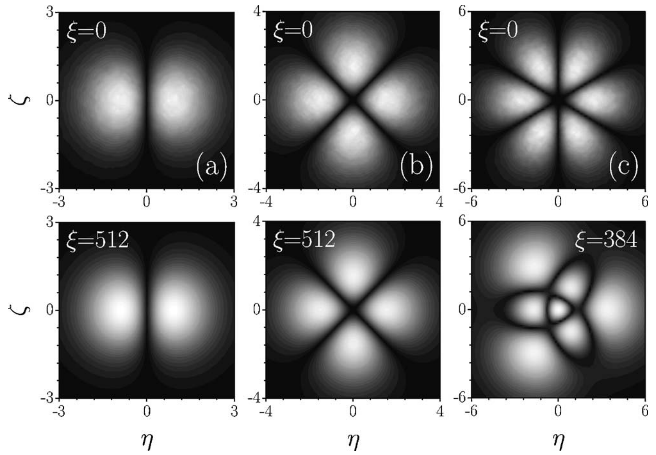

Fig. 3. Propagation dynamics of vector solitons in the presence of white input noise with variance $\sigma_{\text{noise}}^2=0.01$. (a) Dipole soliton at $b_1=3$, $b_2=1.8$, and $d=3$. (b) Quadrupole soliton at $b_1=3$, $b_2=0.9$, and $d=4$. (c) Hexapole soliton at $b_1=3$, $b_2=0.4$, and $d=5$. Field modulus distributions in second components are shown.

ried by the multipole component decreases below a certain threshold, i.e., for $b_2^{\text{low}} \leq b_2 \leq b_2^{\text{cr}}$. The critical value b_2^{cr} increases with d [Figs. 2(c) and 2(f)]. At $d \to 0$, the topological structure of the instability domain becomes complex, so that in some cases we determined b_2^{cr} only starting from a certain minimal d value.

Propagation of perturbed multipole vector solitons is illustrated in Fig. 3. We solved Eqs. (1) by the split-step Fourier method for input conditions $q_{1,2}|_{\xi=0} = w_{1,2}(1+\rho_{1,2})$, where $\rho_{1,2}(\eta,\zeta)$ stand for white noise with the Gaussian distribution and variance σ_{noise}^2. Stable multipole vector solitons retain their structure over indefinitely long distances even in the presence of input noise [Figs. 3(a) and 3(b)]. Similar scenarios were encountered for higher-order solitons. Interestingly, the w_2 component of unstable multipole solitons in strongly nonlocal media may undergo noise-induced kaleidoscopic transformations [like those shown in Fig. 3(c)] periodically, almost restoring its input structure [thus in Fig. 3(c), the first restoration of input intensity distribution occurs at $\xi \approx 540$].

Summarizing, we found that in the media with the Helmholtz-type nonlocal response vectorial coupling with the nodeless beam is a necessary condition for stabilization of multipole solitons, which in the scalar case are highly unstable upon propagation. Our findings suggest that the physical nature, and hence the spatial shape of the nonlinear response function, is a crucial factor for stability of higher-order solitons.

This work was partially supported by the Government of Spain through grant TEC2005-07815/MIC and CONACyT under project 46522. Y. Kartashov's e-mail address is Yaroslav.Kartashov@icfo.es.

References

1. M. Peccianti, K. A. Brzdakiewicz, and G. Assanto, Opt. Lett. **27**, 1460 (2002).
2. M. Peccianti, C. Conti, and G. Assanto, Opt. Lett. **30**, 415 (2005).
3. F. Derrien, J. F. Henninot, M. Warenghem, and G. Abbate, J. Opt. A: Pure Appl. Opt. **2**, 332 (2000).
4. A. G. Litvak, V. A. Mironov, G. M. Fraiman, and A. D. Yunakovskii, Sov. J. Plasma Phys. **1**, 31 (1975).
5. A. V. Mamaev, A. A. Zozulya, V. K. Mezentsev, D. Z. Anderson, and M. Saffman, Phys. Rev. A **56**, R1110 (1997).
6. W. Krolikowski, O. Bang, J. J. Rasmussen, and J. Wyller, Phys. Rev. E **64**, 016612 (2001).
7. J. Wyller, W. Krolikowski, O. Bang, and J. J. Rasmussen, Phys. Rev. E **66**, 066615 (2002).
8. C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
9. W. Krolikowski, O. Bang, N. I. Nikolov, D. Neshev, J. Wyller, J. J. Rasmussen, and D. Edmundson, J. Opt. B: Quantum Semiclassical Opt. **6**, S288 (2004).
10. I. A. Kolchugina, V. A. Mironov, and A. M. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 333 (1980) [JETP Lett. **31**, 304 (1980)].
11. X. Hutsebaut, C. Cambournac, M. Haelterman, A. Adamski, and K. Neyts, Opt. Commun. **233**, 211 (2004).
12. Z. Xu, Y. V. Kartashov, and L. Torner, Opt. Lett. **30**, 3171 (2005).
13. N. I. Nikolov, D. Neshev, W. Krolikowski, O. Bang, J. J. Rasmussen, and P. L. Christiansen, Opt. Lett. **29**, 286 (2004).
14. A. Dreischuh, D. N. Neshev, D. E. Petersen, O. Bang, and W. Krolikowski, Phys. Rev. Lett. **96**, 043901 (2006).
15. V. A. Mironov, A. M. Sergeev, and E. M. Sher, Dokl. Akad. Nauk SSSR **260**, 325 (1981) [Sov. Phys. Dokl. **26**, 861 (1981)].
16. S. Lopez-Aguayo, A. S. Desyatnikov, Y. S. Kivshar, S. Skupin, W. Krolikowski, and O. Bang, arXiv.org e-Print archive, nlin.PS/0512053 December 20, 2005, http://arxiv.org/abs/nlin/0512053.
17. M. Mitchell, M. Segev, and D. N. Christodoulides, Phys. Rev. Lett. **80**, 4657 (1998).
18. E. A. Ostrovskaya, Y. S. Kivshar, D. V. Skryabin, and W. J. Firth, Phys. Rev. Lett. **83**, 296 (1999).
19. J. J. Garcia-Ripoll, V. Perez-Garcia, E. A. Ostrovskaya, and Y. S. Kivshar, Phys. Rev. Lett. **85**, 82 (2000).
20. W. Krolikowski, E. A. Ostrovskaya, C. Weilnau, M. Geisser, G. McCarthy, Y. S. Kivshar, C. Denz, and B. Luther-Davies, Phys. Rev. Lett. **85**, 1424 (2000).
21. T. Carmon, C. Anastassiou, S. Lan, D. Kip, Z. H. Musslimani, M. Segev, and D. N. Christodoulides, Opt. Lett. **25**, 1113 (2000).
22. A. Desyatnikov, D. Neshev, E. Ostrovskaya, Y. S. Kivshar, W. Krolikowski, B. Luther-Davies, J. J. Garcia-Ripoll, and V. Perez-Garcia, Opt. Lett. **26**, 435 (2001).
23. D. Neshev, G. McCarthy, W. Krolikowski, E. A. Ostrovskaya, G. F. Calvo, F. Agullo-Lopez, and Y. S. Kivshar, Opt. Lett. **26**, 1185 (2001).
24. A. S. Desyatnikov, D. Neshev, E. A. Ostrovskaya, Y. S. Kivshar, G. McCarthy, W. Krolikowski, and B. Luther-Davies, J. Opt. Soc. Am. B **19**, 586 (2002).
25. J. Yang and D. E. Pelinovsky, Phys. Rev. E **67**, 016608 (2003).