Surface vortex solitons

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Abstract: We predict the existence of vortex solitons supported by the surface between two optical lattices imprinted in Kerr-type nonlinear media. We find that such surface vortex solitons can exhibit strongly noncanonical profiles, and that their salient properties are dictated by the location of the vortex core relative to the surface. A refractive index modulation forming the optical lattices at both sides of the interface yields complete stability of the vortex solitons in wide domains of their existence, thus introducing the first known example of stable topological solitons supported by a surface.

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Vortex solitons, i.e. localized excitations which carry screw topological phase dislocations in nonlinear materials, play a central role in many branches of science, including superfluids, plasmas, Bose-Einstein condensates, and nonlinear optics (for recent reviews, see, e.g., [1,2]). Bright vortex solitons are typically prone to azimuthal modulation instabilities that cause their self-splitting into ground-state solitons which fly off the original state. Only a few examples of stable vortex solitons in uniform nonlinear materials have been found to the date, namely vortex solitons in materials with competing nonlinearities, in dissipative systems, and, more recently, in nonlocal media [3-5].

Spatial modulation of the parameters describing light propagation in nonlinear media can have a strong stabilizing action on nonlinear vortices. Thus, it was shown in Refs. [6-9] that stable vortex solitons exist in discrete lattices, e.g. two-dimensional arrays of weakly coupled waveguides. This proposal was verified theoretically for vortex solitons in continuous lattices imprinted in cubic and saturable nonlinear materials [10,11]. Lattice vortex solitons (both off-site and on-site) have been observed later in photorefractive crystals [12,13], where periodic lattices can be induced optically, as predicted in [14] and experimentally demonstrated in [15-18]. Stable vortex solitons were also found in higher-order spectral bands of periodic optical lattices [19,20], in three-dimensional lattices [21,22], in radial Bessel lattices [23-25], in lattices imprinted in materials with quadratic nonlinearities [26] and in photonic crystals [27,28]. Remarkably, vortex solitons can be strongly asymmetric in symmetric lattices [29] and in anisotropic lattices [30].

A new important possibility, never addressed to date to our knowledge, is the existence of vortex solitons at the surface between two different materials. Surface waves were studied in several areas of solid-state physics, as well as in nonlinear and in near-field optics (see, e.g., [31-35]). Nonlinearity may drastically alter the refraction scenario for optical beams and may result in bistability and transition between the regimes of total internal reflection and complete transmission [36,37]. Optical surface waves were observed in photorefractive materials with diffusion nonlinearity [38] and at the interface of uniform and layered media, including photonic crystals [39]. Waveguide arrays fabricated with currently available technologies allow formation of solitons at the interface between optical lattices, a concept put forward recently in Refs. [40,41]. Such surfaces support different types of solitons, including gap surface solitons [42]. Discrete surface solitons [43] were observed very recently in a landmark experiment conducted in waveguide arrays made in AlGaAs.

In this paper we report on the existence and properties of vortex solitons supported by the interface of two different optical lattices imprinted in Kerr-type focusing nonlinear media. To the best of our knowledge, existence of surface vortex solitons has not been reported so far, not even for interfaces of uniform media, thus our findings constitute the first known example of stable topological soliton supported by a surface. We find that surface vortex solitons exhibit strongly asymmetric profiles and noncanonical phase distributions [44,45]. Importantly, we find that because of the effects introduced by lattices forming the interface, surface vortex solitons are completely stable in wide domains of their existence. We reveal the relation existing between the interface parameters and the existence domains of surface vortex solitons.

We consider propagation of laser radiation at the interface between two periodic lattices imprinted in focusing media with Kerr-type saturable nonlinearity, described by the nonlinear Schrödinger equation for the dimensionless complex amplitude of the light field q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) - \frac{q|q|^2}{1+S|q|^2} - R(\eta,\zeta)q. \qquad (1)$$

In Eq. (1) the transverse η, ζ and longitudinal ξ coordinates are scaled in terms of the beam width and the diffraction length, respectively, and S is the saturation parameter. The function $R(\eta,\zeta) = \delta p H(\eta) + (p/4)[1-\cos(\Omega\eta)][1-\cos(\Omega\zeta)]$ stands for the transverse refractive index profile, where p is the depth of the lattice, Ω is its frequency, the function $H(\eta) \equiv 0$ for $\eta \le 0$, and $H(\eta) \equiv 1$ for $\eta > 0$, and δp characterizes the height of the step in the constant refractive index. The profile of such lattice is shown in Fig. 1(c). We assume that the depth of the periodic refractive index modulation and the height of the refractive index step at $\eta = 0$ are small compared with the unperturbed refractive index. We also assume that they are of the order of the nonlinear contribution due to the Kerr effect.

Such refractive index landscapes can be either fabricated technologically (in particular by ion implantation) in suitable saturable nonlinear media, or they might be induced optically in photorefractive crystals. In the latter case the periodic part of lattice might be created by interfering four plane waves, while a non-uniform incoherent background illumination of the crystal can produce a sharp step in the refractive index at $\eta = 0$. This can be achieved when the propagation direction for the background illumination is perpendicular to the ξ-axis, so that reshaping of the background wave due to diffraction is negligible over tenths of lattice periods in the (η, ζ) plane and, hence, the corresponding optically induced refractive index change can be accurately described by a step-like function $H(\eta)$. Standard techniques [15-18] based on vectorial interactions can be employed to observe soliton formation.

It should be pointed out that other types of nonlinear lattice interfaces could potentially be realized in photorefractive crystals by applying different voltages to different crystal parts. In this paper, for the sake of generality, we use the canonical model (1), since it holds for saturable media with technologically imprinted lattice and simultaneously describes main qualitative features of light propagation in optically induced lattices. Equation (1) admits several conserved quantities, including the power or energy flow

$$U = \int\int_{-\infty}^{\infty} |q|^2 \, d\eta d\zeta. \tag{2}$$

We search for spatially localized vortex soliton solutions of Eq. (1) in the following form: $q = [w_{\rm r}(\eta,\zeta) + i w_{\rm i}(\eta,\zeta)]\exp(ib\xi)$, where functions $w_{\rm r}$ and $w_{\rm i}$ represent real and imaginary parts of light field, respectively, and b is the propagation constant. The topological winding number (or vortex charge) m of the complex field q can be defined by the circulation of the gradient of the field phase $\arctan(w_{\rm i} / w_{\rm r})$ around the phase singularity, where the field vanishes. Here we focus on vortex soliton solutions with unit topological charge. Substitution into Eq. (1) yields the system

$$\frac{1}{2}\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right) w_{\rm r,i} - b w_{\rm r,i} + \frac{w_{\rm r,i}(w_{\rm r}^2 + w_{\rm i}^2)}{1 + S(w_{\rm r}^2 + w_{\rm i}^2)} + R w_{\rm r,i} = 0, \tag{3}$$

We solved system (3) numerically with a standard relaxation method. It is apparent that very far from the interface located at $\eta = 0$ the properties of vortex solitons supported by lattice shown in Fig. 1(c) do not differ substantially from the properties of their counterparts in perfectly periodic lattices. This situation changes as soon as the soliton energy concentrates in the lattice sites adjacent to the interface. In this case the internal structure is different for vortices shifted into lattice regions with lower (at $\eta < 0$) or higher (at $\eta > 0$) mean refractive index values.

The simplest situation occurs when vortex phase singularity is located close to the point $\eta = \zeta = 0$. Some representative examples of the profiles of such vortex solitons are depicted in Figs. 1(a) and 1(b). By analogy with perfectly periodic lattice we term such vortex solitons "off-site", since the phase singularity is located between the local lattice maxima. Four main intensity lobes (whose separation is minimal for the off-site case) are clearly resolvable in the vortex profile, but the lobes located at $\eta > 0$ are smaller than those located at $\eta < 0$. Therefore, the vortices become strongly asymmetric, especially for large refractive index steps δp. The phase distributions for asymmetric surface vortex solitons are non-canonical [44,45], in the sense that around a ring whose center coincides with the phase singularity, the phase does not increase linearly, but rather possesses alternating regions of slow (in the vicinity of local soliton intensity maxima) and fast growth (see Fig. 2 where a canonical phase distribution is

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depicted). Increasing δp leads to stronger asymmetry. On the other hand, a growing periodic modulation depth p leads to a stronger localization of the vortex energy near the local lattice maxima.

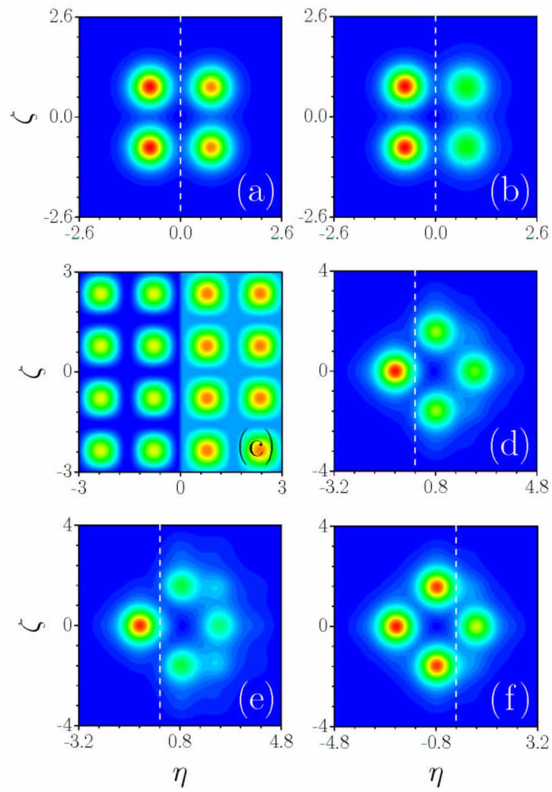

Fig. 1. Field modulus distributions for vortex surface solitons at $b = 8$, $p = 4.5$, $S = 0.05$, and (a) $\delta p = 1$, (b) $\delta p = 4$. (c) Lattice profile at $p = 4.5$ and $\delta p = 1$. Field modulus distributions for the highly asymmetric vortex solitons at $b = 3$, $p = 4$, $S = 0.2$, and (d) $\delta p = 0.8$, (e) $\delta p = 1.3$, (f) $\delta p = 0.6$. In all cases $\Omega = 4$. Vertical dashed lines indicate the location of the interface.

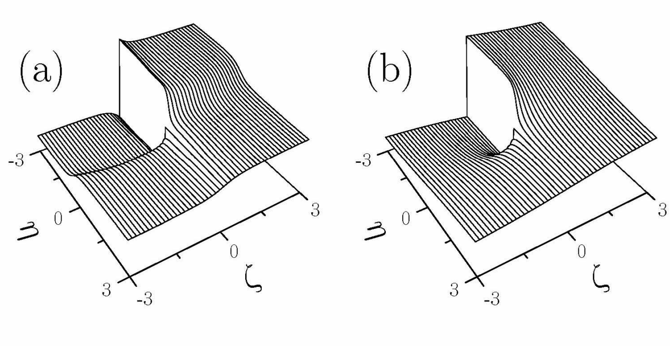

Fig. 2. (a) Phase distribution for surface vortex soliton depicted in Fig. 1(b). (b) Same but for a similar canonical vortex soliton supported by a uniform medium.

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We found that for the fixed δp and p there are lower $b_{\rm low}$ and upper $b_{\rm upp}$ cutoffs for the existence of off-site surface vortex solitons. In contrast to canonical vortices, the ratio $U_{\rm r}/U_{\rm i}$

$$U_{\rm r,i} = \int\int_{-\infty}^{\infty} w_{\rm r,i}^2 d\eta d\zeta \tag{4}$$

is not constant and varies with all the physical parameters, namely b, p, and δp. We found that close to the lower cutoff and the upper cutoff one of the ratios $U_{\rm r,i}$ abruptly tends to zero. The domain of existence for surface vortex solitons is presented in Fig. 3(a). The width of the existence domain in b is maximal for $\delta p \to 0$ (in this case one has $b_{\rm upp} = b_{\rm low} + 1/S$), and *quickly shrinks* with increasing δp.

Off-site surface vortex solitons *cease to exist* above a *critical* value of the step $\delta p_{\rm cr}$. For a fixed δp the width of the existence domain broadens with a decreasing lattice depth p. Though dependencies $U_{\rm r,i}(b)$ are non-monotonic close to the cutoffs, the total energy flow U still is the monotonically increasing function of propagation constant (Fig. 3(b)). The maximal energy flow carried by a surface vortex soliton residing at the interface quickly decreases with increasing δp. We found that close to the lower cutoff surface vortex solitons are typically less localized than near the upper cutoff. This is especially pronounced at small or moderate values of $\delta p \sim 1$, when low-energy vortex solitons expand over several neighboring lattice sites. Moreover, expansion in the region $\eta > 0$ can be much more pronounced than that for $\eta < 0$. Surface vortex solitons with strongly asymmetric shapes at $\delta p \gg 1$ are typically well localized at both cutoffs.

A particularly interesting property of surface vortex solitons is that the critical value of refractive index step $\delta p_{\rm cr}$ *decreases* with increasing lattice depth (see Fig. 3(c)), i.e. *stronger asymmetries* are achieved at interfaces between *shallower* lattices. The relation between the interface parameters and the domains of existence for vortex solitons can be intuitively understood by considering the energy circulation in the vortex. The existence of stationary vortex requires exact balance between the flows of energy through the interface in positive and negative direction of η axis. Such balance is no longer possible when the difference in refractive indices becomes too large. This is manifested in shrinking of the existence domain of surface vortex solitons by increasing the step δp.

One of the central results of this paper is that strongly asymmetric vortex solitons can be made *completely stable* in a substantial part of their existence domain. To rigorously and comprehensively analyze the stability of the whole family of vortex solitons, we searched for perturbed solutions of Eq. (1) in the form $q = (w_{\rm r} + iw_{\rm i} + u_{\rm r} + iu_{\rm i})\exp(ib\xi)$, where $u_{\rm r}(\eta,\zeta,\xi)$, $u_{\rm i}(\eta,\zeta,\xi)$ are real and imaginary parts of a small perturbation $(|u_{\rm r,i}| \ll |w_{\rm r,i}|)$ that can grow with a complex rate δ upon propagation. Linearization of Eq. (1) around stationary solutions $w_{\rm r}(\eta,\zeta)$, $w_{\rm i}(\eta,\zeta)$ obtained from Eqs. (3), at first order of perturbation theory, yields the system of equations

$$\begin{aligned}&\left(\frac{3w_{\rm r,i}^2 + w_{\rm i,r}^2 + S(w_{\rm r}^2 + w_{\rm i}^2)^2}{[1 + S(w_{\rm r}^2 + w_{\rm i}^2)]^2} u_{\rm r,i} + \frac{2w_{\rm r}w_{\rm i}}{[1 + S(w_{\rm r}^2 + w_{\rm i}^2)]^2} u_{\rm i,r}\right) + \\ &+\frac{1}{2}\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right)u_{\rm r,i} - bu_{\rm r,i} + Ru_{\rm r,i} = \pm\frac{\partial}{\partial\xi}u_{\rm i,r}\end{aligned} \tag{5}$$

where we assume that $u_{\rm r,i} \sim \exp(\delta\xi)$. We solved this system numerically in order to find the perturbation profiles and associated growth rates δ. The existence of perturbations having $\mathrm{Re}\,\delta > 0$ implies instability (termed exponential for $\mathrm{Im}\,\delta = 0$, and oscillatory for $\mathrm{Im}\,\delta \neq 0$) of the corresponding stationary solutions $w_{\rm r,i}$. The development of such perturbations should

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result in strong distortion of input intensity distribution with propagation distance ξ . The absence of perturbations with $\mathrm{Re}\,\delta > 0$ reveals linear stability of the corresponding stationary solutions. Addition of small random or regular perturbations into such stable solutions may result only in small oscillations of the field amplitude around the input value.

The outcome of our comprehensive stability analysis for off-site surface vortex solitons performed for various sets of parameters δp and p is summarized in Fig. 4. We found that strongly asymmetric vortex solitons are *stable* in most part of their existence domain at moderate $p \sim 1$ and high depths of periodic modulation. Our calculations reveal two instability bands located near the lower and upper existence cutoffs (Fig. 4(c)). The instability domain located near the upper cutoff is very narrow and not even visible in the plot. The width of the lower instability domain *decreases* with growing δp . The instabilities encountered for asymmetric surface vortex solitons are associated with a complex growth rate δ and, hence, are of oscillatory type. The typical dependencies of the real part of the perturbation growth rate on b are depicted in Figs. 4(a) and 4(b). Note, that increasing the depth of the periodic modulation p results in further reduction of the widths of lower and upper instability domains. Therefore,

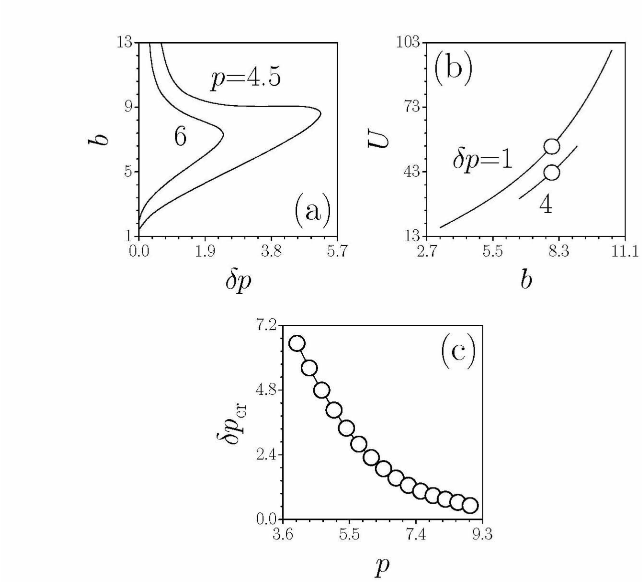

Fig. 3. (a) Domains of existence of surface vortex solitons at the $(\delta p, b)$ plane. (b) Energy flow versus propagation constant at $p = 4.5$. Points marked by circles correspond to solitons shown in Figs. 1(a) and 1(b). (c) Critical value of δp versus lattice depth. In all cases $\Omega = 4$, $S = 0.05$.

the important result put forward by these findings is that despite the strong asymmetry of the surface vortex solitons, they can be still completely stable in the wide parameter domains. We attribute such stabilization to the periodic refractive index modulation existing at both sides of the interface. Namely, the mechanism behind stabilization of surface vortex solitons is expected to be similar to that for usual on- and off-site vortex solitons in uniform periodic lattices. This is consistent with the fact that an interface between a lattice and a uniform medium also supports surface vortex solitons, but we found all of them to be always strongly unstable on propagation.

To confirm results of the above linear stability analysis we solved Eq. (1) directly with the input conditions $q|_{\xi=0} = (w_{\mathrm{r}} + i w_{\mathrm{i}})(1+\rho)$, where $\rho(\eta,\zeta)$ is a random function with a Gaussian distribution and variance $\sigma^2_{\mathrm{noise}}$. A split-step Fourier method was used to perform

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the simulations. The accuracy of the results was tested by doubling the integration window and the number of grid points, and verifying that identical results were obtained. Typically grids with up to 1024×1024 points and longitudinal step $d\xi = 0.0025$ were used. The numerical simulations always confirmed the predictions by the linear stability analysis. Asymmetric surface vortex solitons belonging to the stable domain retain their input structure for huge distances, far exceeding any experimentally achievable crystal lengths (see Fig. 5(a)), while unstable representatives of the off-site vortex families decay via progressively growing oscillations of their intensity lobes (Fig. 5(b)).

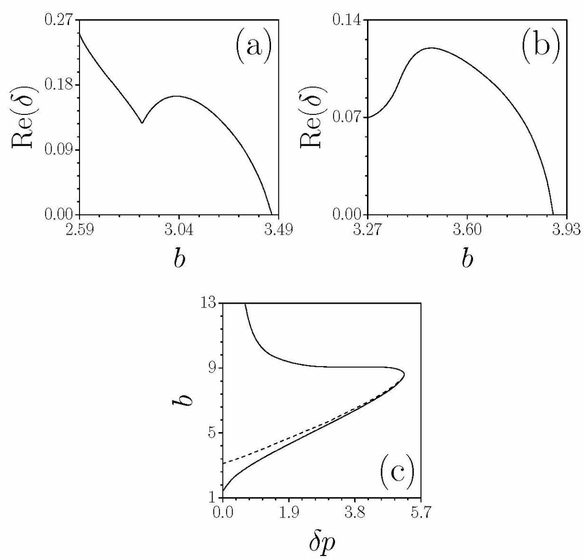

Fig. 4. Real part of perturbation growth rate versus propagation constant for $p = 4.5$ at $\delta p = 0.5$ (a) and $\delta p = 1$ (b). (c) Stability and instability domains for surface vortex solitons on $(\delta p, b)$ plane at $p = 4.5$. Vortex solitons exist in the region between upper and lower solid lines. They are stable in the region above dashed line and oscillatory unstable in the region between dashed and lower solid lines. In all cases $\Omega = 4$, $S = 0.05$.

In addition, it is worth noticing that besides the simplest off-site surface vortex solitons we also found a variety of other asymmetric vortex solitons families. Representative examples of the on-site vortices that reside mainly at either side of the interface are shown in Figs. 1(d) – 1(f). Notice that upon searching for such solitons we translated the lattice by π / Ω in the vertical direction for convenience. We found that on-site vortex solitons are much more sensitive to variations in the height of refractive index step δp and typically require $\delta p < p$, so that the existence domain substantially departs from that of their off-site counterparts. Nevertheless, on-site vortex solitons also exhibit strongly asymmetric shapes (see, e.g. Fig. 1(e)), and can be made completely stable in suitable domains of their existence. Finally, we would like to remark that we also found that lattices with *defocusing nonlinearity* can support asymmetric surface vortex solitons as well.

We thus conclude stressing that we have reported, for the first time to our knowledge, the existence of surface vortex solitons. We have found that such vortex solitons exist only when the refractive index at the surface does not exceed a critical value, which is dictated by the depth of the lattices. The surface vortex solitons can be made stable and robust under proper conditions, and exhibit strongly asymmetric and noncanonical nature.

Here we addressed the interface between two distinct periodic lattices with focusing nonlinearity, but results have implications for other physical settings, including lattices made in defocusing media. Also, we believe that our findings reported here motivate the search of other types of topological solitons supported by surfaces.

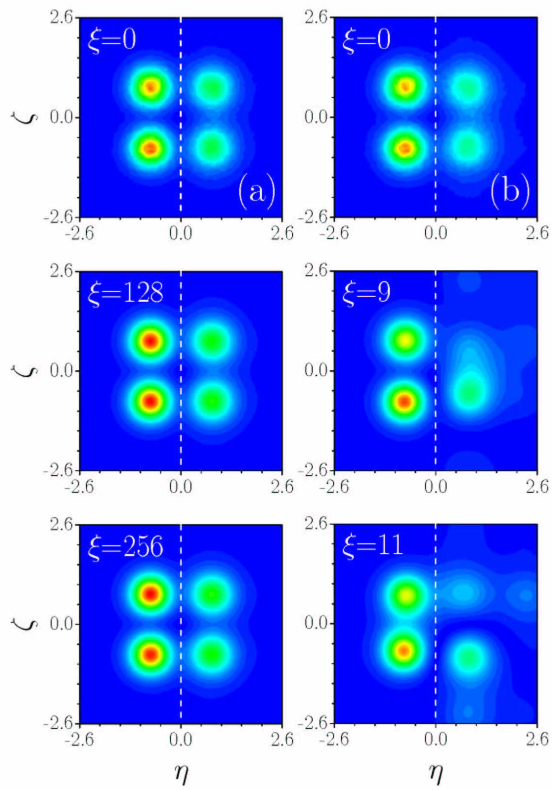

Fig. 5. Propagation dynamics of surface vortex solitons with $b = 8$ (a) and $b = 6.65$ (b) at $p = 4.5$ and $\delta p = 4$ in the presence of white input noise with the variance $\sigma^2_{\text{noise}} = 0.01$. Vertical dashed lines indicate interface position. In all cases $\Omega = 4$, $S = 0.05$.

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