Symmetry breaking in small rotating clouds of trapped ultracold Bose atoms

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(Received 31 October 2006; revised manuscript received 7 May 2007; published 25 July 2007)

We study the signatures of rotational and phase symmetry breaking in small rotating clouds of trapped ultracold Bose atoms by looking at rigorously defined condensate wave function. Rotational symmetry breaking occurs in narrow frequency windows, where energy degeneracy between the lowest energy states of different total angular momentum takes place. This leads to a complex condensate wave function that exhibits vortices clearly seen as holes in the density, as well as characteristic local phase patterns, reflecting the appearance of vorticities. Phase symmetry (or gauge symmetry) breaking, on the other hand, is clearly manifested in the interference of two independent rotating clouds.

DOI: 10.1103/PhysRevA.76.013625 PACS number(s): 03.75.Hh, 05.30.Jp

I. INTRODUCTION

Symmetry breaking in finite systems has been a subject of intensive debate over the years in physics, in general (cf. Ref. [1]), and in physics of ultracold gases, in particular. For Bose-Einstein condensates two symmetries play a particular role: U(1) phase symmetry, and SU(2) [or SO(3)] rotational symmetry. In the large N limit, one breaks these symmetries by hand, as proposed originally by Bogoliubov [2]. Thus, the accurate way to deal with macroscopic Bose-Einstein condensates is by the use of a classical field, also called an order parameter (OP), or the wave function (WF) of the condensate. This function is a single particle wave function, which is the solution of the Gross-Pitaevskii equation valid in the mean field approximation, that characterizes the system in a proper way [3]. It has an arbitrary, but fixed global phase, and for rotating systems with more than one vortex, it exhibits fixed, but arbitrarily oriented vortex arrays. For dilute ultracold Bose gases (i.e., when $n|a|^3 \ll 1$ [4], where n is the density and a is the s-wave scattering length) the mean field, or Bogoliubov approach is capable to reproduce very well the main properties of a Bose-Einstein condensate (BEC), despite the fact that for finite fixed N and total angular momentum L, which are both constants of the motion, mean field theory cannot be exact. This observation has stimulated a lot of discussion about the nature of the phase in BEC [5–8], and about the particle-conserving Bogoliubov approach [9]. The modern point of view (for a recent discussion see [10]) implies that two BEC with fixed N, each one (and therefore completely undefined global phases) will, nevertheless, produce a well-defined interference pattern of fringes as a result of the measurement in only one shot [arrangement compatible with the theoretically predicted n-particle-correlation function (n-PCF)] [6]. This implies that the output of the measurement is equivalent to sampling according to n-PCF and not to the density. Moreover, the breaking of rotational symmetry should occur in large rotating clouds in a similar way: a pure L state would show in a time-of-flight (TOF) experiment, a definite interference pattern accurately represented by the n-PCF, and different from a circular symmetric profile of the single particle density. Observation of such interference pattern would be a test of the meaning assigned to the measurement. Unfortunately, for large N systems, the total angular momentum of the stationary states is not well defined, and there is no qualitative difference between density and n-PCF: both of them typically exhibit vortex arrays. For small rotating clouds, however, the situation is different, as we have shown in Ref. [11]. Typically, the ground states (GS) are pure-L states for most values of the rotating frequency Ω, producing circularly symmetric single particle densities. Only in the very narrow windows of frequencies, where the GS is a linear combination of degenerated states with different L, vortex arrays can be obtained. An arbitrarily small symmetry breaking deformation of the trap potential leads then to the appearance of symmetry breaking vortex arrays visible both in the density, and in the pair correlations. Outside these windows, in the regime of well-defined L GS, small systems would provide a suitable test for the meaning of the measurement distinguishing between the density, and the pair-correlation output. We will address a proposal for a corresponding experiment in Sec. III.

Our main goal in this article is the analysis of the effects of symmetry breaking in small rotating clouds of trapped ultracold Bose atoms in more depth. We achieve it by introducing the condensate WF in a rigorous way, as an eigenfunction of the one body density matrix operator (OBDM) obtained from the exact diagonalization method, and corresponding to the largest eigenvalue. This wave function shares the advantages of both formalisms. It gains the intuitive picture provided by the OP, typical of mean field theories, and at the same time, it is obtained rigorously from the exact GS, and not from a somehow arbitrary choice of a variational ansatz. Such definition of the OP has been introduced in classical papers on *off-diagonal long-range order* [12]. It has, however, rarely been used, since its application requires the knowledge of the full many-body WF, or at least of the exact OBDM (for rare exceptions, see Ref. [13]). Here, we apply this method to the rotating gas, using exact numerically calculated GS and OBDM for few atom systems. In the regime of relatively low rotation frequency (but still inside the lowest Landau level, see Sec. II), where the degree of condensation is high, and some vortices appear distributed in an ordered array, this scalar field plays the role of a genuine OP. It loses, on the other hand, its ability to represent the system as Ω approaches the melting point, where the prediction is that the vortex lattice disappears, and the system turns, for large systems, into a Laughlin liquid [14].

To prove the validity of the OP as a complement of the numerical analysis we address two questions. On the one hand, we identify possible states with vortices, and obtain *local* phase characteristics of the condensate wave function (reflecting quantized circulation of vortices). On the other hand, with such calculated OP we reproduce the density and interference patterns for two condensed clouds, and shed light on the discussion of the origins of symmetry breaking in finite mesoscopic systems. In addition, we provide an unambiguous definition of the degree of condensation for small systems.

This paper is organized as follows. In Sec. II we describe the model used in our calculations, the way in which we obtain the OP as the macro-occupied WF, and the regime in which it is a valid approximation of the exact GS. In Sec. III we show our main results and address the question concerning the meaning of measurement. Finally, in Sec. IV we draw our conclusions.

II. THE MODEL: MACRO-OCCUPIED WAVE FUNCTION

We consider a two-dimensional system of few Bose atoms confined in a parabolic trap rotating around the z axis, and submitted to a stirring laser that creates a slight anisotropy in the xy plane. The rotation frequency Ω is strong enough to assume the lowest Landau level (LLL) regime with atoms interacting via contact forces. In the second quantized form the Hamiltonian of the system projected onto the LLL in the rotating reference frame is described by [11]:

$$\hat{H} = \alpha\hat{L} + \beta\hat{N} + \hat{V} + \hat{V}_p \equiv \hat{H}_0 + \hat{V} + \hat{V}_p, \tag{1}$$

where $\alpha=\hbar(\omega_\perp-\Omega)$, $\beta=\hbar\omega_\perp$, and $\omega_\perp$ being the trap frequency. $\hat{L}$ and $\hat{N}$ are the total z-component angular momentum and particle number operators, respectively. The contact interaction term is given by the operator

$$\hat{V} = \frac{1}{2}\sum_{m_1m_2m_3m_4} V_{1234} a^\dagger_{m_1} a^\dagger_{m_2} a_{m_4} a_{m_3}, \tag{2}$$

where the matrix elements are given by

$$V_{1234} = \langle m_1 m_2 | V | m_3 m_4\rangle = \frac{g}{\lambda^2\pi}\frac{\delta_{m_1+m_2,m_3+m_4}}{\sqrt{m_1!\,m_2!\,m_3!\,m_4!}}\frac{(m_1+m_2)!}{2^{m_1+m_2+1}}, \tag{3}$$

where g is the interaction coefficient that approximates the potential of the Van der Waals forces in the very dilute limit, $\lambda=\sqrt{\hbar/2M\omega_\perp}$, and M is the atomic mass. The last term in Eq. (1) is the anisotropic term that mimics the stirring laser and is given by $V_p=A\sum_{i=1}^N(x_i^2-y_i^2)$ or in second quantized form by

$$\hat{V}_p = \frac{A}{2}\lambda^2\sum_m \sqrt{m(m-1)}a^\dagger_m a_{m-2} + \sqrt{(m+1)(m+2)}a^\dagger_m a_{m+2}. \tag{4}$$

We assume this term to be a small perturbation of the system, thus, $A\lambda^2/[2\hbar(\omega_\perp-\Omega)] \ll 1$. In the previous equations, we considered the Fock-Darwin (FD) single particle set of functions and the operators a_m and $a^\dagger_m$ [15] to represent the many-body WF's and the terms of the Hamiltonian. These set of functions are the solutions of $\hat{H}_0$ in Eq. (1) in the LLL regime and are given by $\varphi_m(\vec{r})=e^{im\theta}r^m e^{-r^2/2}/\sqrt{\pi m!}$ in units of λ, m being the single particle angular momentum ranging from $m=0,1,\ldots$. In Eqs. (2) and (4), $a^\dagger_m$ and a_m creates and annihilates a boson with single particle angular momentum m, respectively. The conditions for validity of LLL regime are given by $(N-1)g/4\pi \ll (1+\Omega/\omega_\perp)$ and $(1-\Omega/\omega_\perp) \ll (1+\Omega/\omega_\perp)$, where λ and $\hbar\omega_\perp$ are taken as units of length and energy, respectively, meaning that the interaction and the kinetic contributions to the energy per particle are smaller than the energy gap between Landau levels which is given by $\hbar(\omega_\perp+\Omega)$. This means that for $g=1$ and N less than about 10 particles the LLL assumption is valid down to frequencies significantly lower than the critical value Ω_c, where the nucleation of the first vortex takes place.

To obtain the single particle maccro-ocupied WF we proceed as follows. In the first step we generate a vortex state turning Ω around the critical frequency within a narrow window, where energy degeneracy takes place among eigenfunctions of $\hat{H}_0+\hat{V}$, and adjust the anisotropy [A in Eq. (4)] to obtain the appropriate linear combination. A necessary condition to generate vortices is given by the presence, in the linear combination, of L states with L and $L\pm 2n$ where n is an integer, as can be inferred from Eq. (4). The anisotropic term must be extremely small in such a way that the GS is extremely similar to the appropriate combination of the degenerated eigenstates of the symmetric Hamiltonian. To be more specific, $AL/2$ must be larger than the energy differences of the L states involved in the linear combination, and lower than the gaps to their excited states. The output is quite robust against variations inside these limits. Once the exact vortex GS is obtained, information about the mostly occupied FD function is also available. However, typically, this is not necessarily the best option to determine the OP. Other linear combinations of FD functions can have a larger occupation. The proper way to know if there is a "macro-occupied" single particle wave function in the ground state $|\phi_{GS}\rangle$ is to look at the eigenvalues and eigenvectors of the OBDM [4,12]. That is to say, one must solve the eigenvalue equation

$$\int d\vec{r}'\, n^{(1)}(\vec{r},\vec{r}')\psi_k^*(\vec{r}') = n_k\psi_k^*(\vec{r}), \tag{5}$$

where

$$n^{(1)}(\vec{r},\vec{r}') = \langle\phi_{GS}|\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}')|\phi_{GS}\rangle, \tag{6}$$

with $\hat{\Psi}=\sum_{m=0}^\infty \varphi_m(\vec{r})a_m$ being the field operator. If there exists a relevant eigenvalue $n_1 \gg n_k$ for $k=2,3,\ldots m_0+1$, then

$$\sqrt{n_1}\psi_1(\vec{r})e^{i\phi_1} \tag{7}$$

plays the role of the OP of the system, where ϕ_1 is an arbitrary global constant phase. Here, m_0 is an integer equal to the largest total angular momentum L involved in the expansion of the GS on L eigenfunctions. The OP may be expanded in the form $\psi_1(\vec{r})=\sum_{m=0}^{m_0}\beta_{1m}\varphi_m(\vec{r})$, where φ_m are the FD functions. Notice that m labels the single particle angular momentum from $m=0,1,\ldots,m_0$, whereas $k=1,2,\ldots,m_0+1$ is a label that distinguishes between the eigenfunctions of the OBDM.

An alternative and perhaps even more appropriate single particle basis is determined by the functions $\psi_k(\vec{r})$. One can define a set of canonical creation and annihilation operators for them, $\hat{b}_k^\dagger=\int d\vec{r}\psi_k(\vec{r})\hat{\Psi}^\dagger(\vec{r})$, with $\hat{b}_k$ being the Hermitian conjugate of $\hat{b}_k^\dagger$, and the new Fock (occupation number) many body basis $|n_1\rangle\otimes|n_2\rangle\otimes\ldots$. The macro-occupied mode contains on average n_1 atoms, but this number fluctuates. This implies that atom number fluctuations between the macro-occupied mode (condensate) and the rest of the modes (that could be regarded as phonon modes, quasi-particles) will tend to reduce the fluctuations of the phase. A natural consequence of this observation is to expect that a very fine approximation of the GS is given by the coherent state $|\alpha_1\rangle$ such that $\hat{b}_1\;|\alpha_1\rangle=\sqrt{n_1}e^{i\phi_1}\;|\alpha_1\rangle$. If n_k for $k=2,3,\ldots$ are smaller than n_1 we may neglect them, and approximate the many body WF by $|\alpha_1\rangle\otimes|0_2\rangle\otimes|0_3\rangle\otimes\ldots$. This representation implies that the next simplifying step would be the representation of the GS by a classical field containing all the involved coherent states $|\alpha_k\rangle$, $k=1,\ldots m_0+1$ as $\Psi(\vec{r})=\sum_{k=1}^{m_0+1}\sqrt{n_k}\psi_k e^{i\phi_k}$ with random phases. Calculation of quantum mechanical averages would then in principle require averaging over random phases, which makes this approach technically difficult.

As long as the exact GS is a state with well-defined angular momentum (a pure L state solution of the Hamiltonian for Ω far from the narrow windows, where the slight anisotropy has no effect), it is easy to demonstrate that the FD functions are the eigenstates of Eq. (5) and the eigenvalues n_k are the occupations usually used in literature, or in other words, that ψ_k and φ_m constitute the same basis. However, for the vortex states the eigenfunctions of Eq. (5) are linear combinations of the FD functions as previously stated, and the macro-occupied function ψ_1 that represents the whole state has expected single particle angular momentum given by $\hbar\tilde{m}=\int\psi_1^*(\vec{r})\hat{L}\psi_1(\vec{r})d\vec{r}=\sum_{m=0}^{m_0}|\beta_{1m}|^2\hbar m$.

A convenient definition of the degree of condensation, which is sensitive to the loss of macro-occupation is given by

$$c=\frac{n_1-\tilde{n}}{N}, \tag{8}$$

where N is the total number of atoms, whereas $\tilde{n}$ is the mean occupation of the rest, that is $\frac{1}{m_0}\sum_{k=2}^{m_0+1}n_k$. The usual definition given by n_1/N for $N\gg1$ is not appropriate for small systems as the total number of implied states (m_0+1) is now a small number, and so even in the absence of condensation with all levels equally occupied, such definition implies a condensate fraction of a few percent. Equation (8), on the other hand, approaches zero for equal occupations, as one would expect.

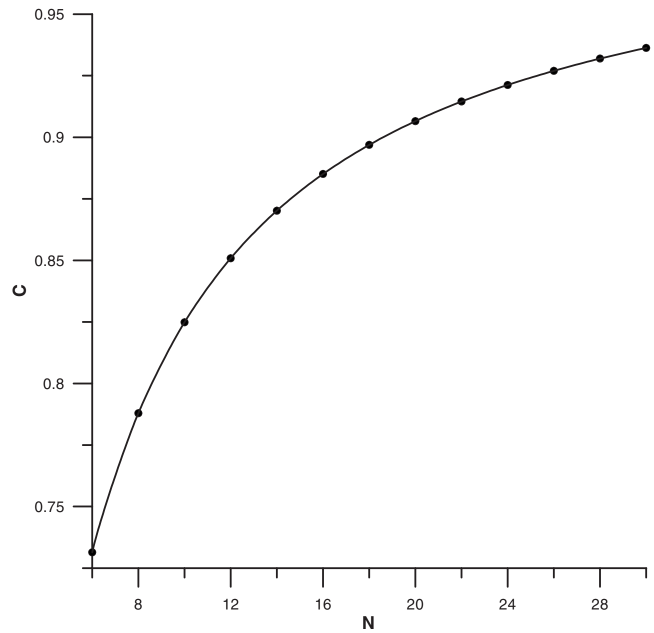

FIG. 1. Degree of condensation [see Eq. (8)] as a function of the number of atoms N, for the state with $L=N$.

III. RESULTS

In what follows, we show some results that confirm how convenient it is to represent the whole system by ψ_1 at certain values of Ω. First of all, we plot in Fig. 1 the degree of condensation as a function of N, for $L=N$. Clearly c approaches 1 rapidly as N grows and is $\geq80\%$ already for 8–9 atoms. As a general result, for vortex states, n_1 is always larger than the occupation of the most important FD state within the exact GS. In addition, ψ_1 provides a nonambiguous way to characterize vortices, not only showing dimples in the density profile, but also indicating the position of each one by the change of the phase $S(\vec{r})$ in $\psi_1(\vec{r})=|\psi_1(\vec{r})|e^{iS(\vec{r})}$ by multiples of 2π, when moving around each vortex. In Figs. 2 and 3 for $N=6$ atoms, we show for three different values of Ω, where degeneracy takes place, the comparison between the densities of the exact GS and the ψ_1 function (Figs. 2 and 3), as well as the contour map of the phase $S(\vec{r})$ of ψ_1 (Fig. 3 only). In the first case, Fig. 3(a), the GS contains two well-defined vortices that appear in a clearer way in the OP, as it excludes the noncondensed part that smears the structure of the GS. The same picture is shown in Fig. 3(b) where four vortices become visible. In the second case, the map of the phase not only localizes vortices with one unit of quantized circulation, but also indicates that incipient vortices are growing at the edge of the system. In the last case Fig. 3(c), a sixfold symmetry is obtained in the GS not attached in this case to vortices, but to a mixed structure of dimples and bumps different from the vortex array shown in the OP. The structure shown in the density of the GS is a precursor of the

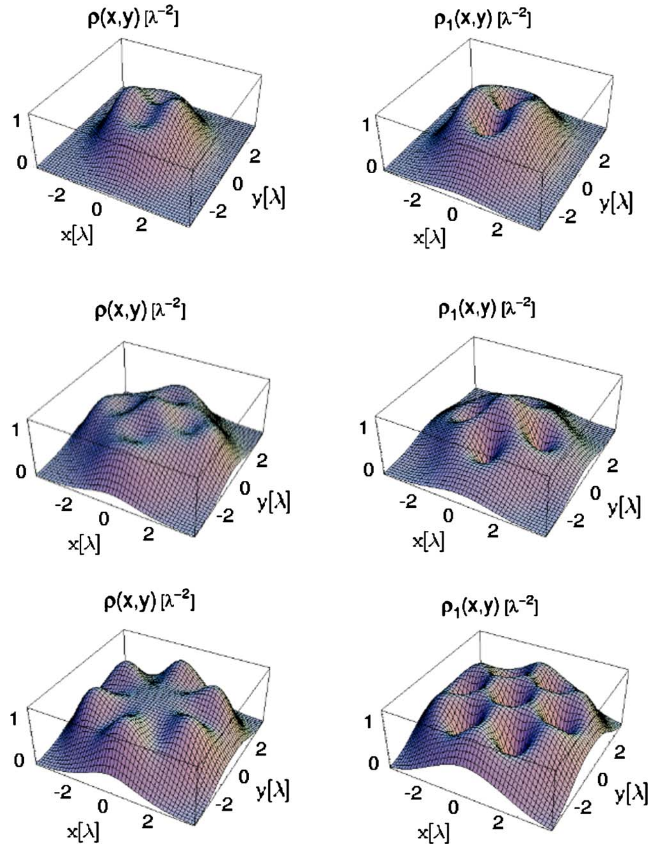

FIG. 2. (Color online) For $N=6$ each row shows three-dimensional plots of the density of the GS $[\rho(x,y)]$ and the ψ_1 function $[\rho_1(x,y)]$, respectively. The first row shows a two-vortex structure at $\Omega/\omega_\perp=0.941$ (where degeneracy between $L=10$, 12, and 14 takes place). The second row shows a four-vortex structure, $\Omega/\omega_\perp=0.979$ (degeneracy between $L=20$, 22, and 24). The third row shows a sixfold structure, $\Omega/\omega_\perp=0.983$ (degeneracy between $L=24$, 26, 28, and 30). In all cases $\omega_\perp=g=1$ in units of λ, and $u=\hbar\omega_\perp$.

Wigner type structure observed for few atoms in the pair correlation function of the Laughlin state (with angular momentum $L=30$, the last angular momentum included in the expansion) [11].

The degree of condensation as defined in Eq. (8) decreases as 0.343, 0.192, and 0.015 from Figs. 3(a) to 3(c). The order in vortices and disorder in atoms evolves to order in atoms. To test the validity of the identification of the order parameter with that of a coherent state, we calculated the fluctuations of n for the cases considered in Fig. 2. We obtained $\Delta n=2.01, 1.32$, and 1.25 in qualitative agreement with the values of $\sqrt{n}$ (the fluctuations of real coherent states with the same expected occupation) given by $1.52, 1.15$, and 0.966, respectively. This result suggests even better localization of the phase than for coherent states, and thus justifies our interference calculation given below. As Ω approaches the frequency of the trap, the occupations tend to equalize (or $n_1=\tilde{n}$) and in the Laughlin state, where n_k are the FD occupations (since it is a pure L state), the degree of condensation tends to zero.

Finally, we show the interference pattern produced by the overlap of two initially independent condensates represented

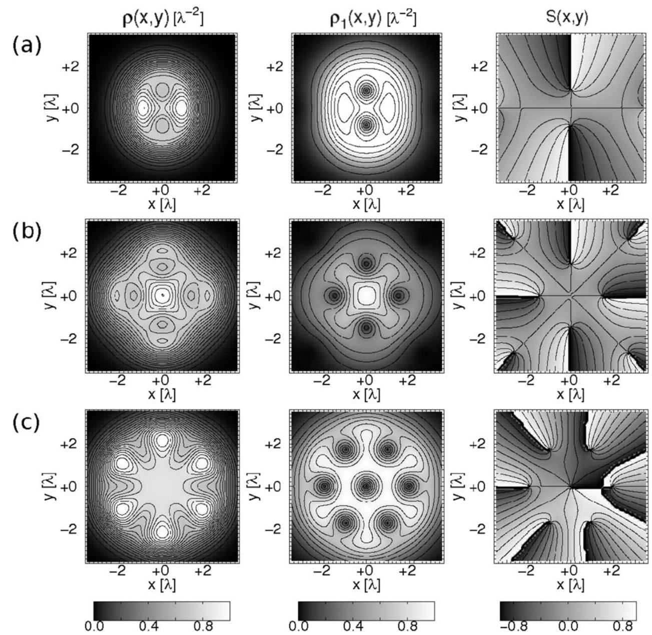

FIG. 3. For $N=6$ each row shows the contour plots of the densities analyzed in Fig. 2, as well as the map of the phase $S(\vec{r})$ (see text). The first, second, and third rows show a twofold, fourfold, and sixfold structure, respectively.

by ψ_1 functions. This study is motivated by an increasing amount of recent work revealing the possibility of obtaining very detailed experimental information on the interference pattern produced not only during the overlap of two or more independent condensates [16], but also within a unique condensate [17].

The idea underlying our approach is the following: we represent the two independent condensates which we call a and b by their macroscopic occupied function ψ_a and ψ_b, respectively. By this we mean that the condensates are in two coherent states $|\alpha_a\rangle$ and $|\alpha_b\rangle$ from which we know their OP, except for their constant phases ϕ_a and ϕ_b [see Eq. (7)] [18]. At time $t=0$, the condensates are separated by a distance d and the traps are switched off. The time evolution of the system is obtained [once the transformation to the laboratory frame of reference is performed, multiplying the functions by $\exp(-i\Omega t\hat{L}_z)$] in three steps: First, the Fourier transform of the total OP (the sum of the two contributions) is performed. Then, the time evolution of the Fourier components by multiplying them by exponentials of the type $\exp(i\hbar k^2 t/2m)$ is realized; this step is done assuming that during the TOF the interactions are irrelevant. Finally, in the third step, inverse Fourier transformation is performed, and the density of the total WF is calculated. The results are shown in Fig. 4 where three different times are considered. Fortunately, the uncertainty about the phase relation $\phi=\phi_a-\phi_b$ is not important in the case considered, as only two terms are involved, and a change of the relative phase produces only a global shift of the interference pattern. Note that our calculation pertains to a single shot experiment, and not to the average of several shots as considered in Ref. [19].

Before closing this section, we would like to emphasize that systems of small number of particles provide new pos-

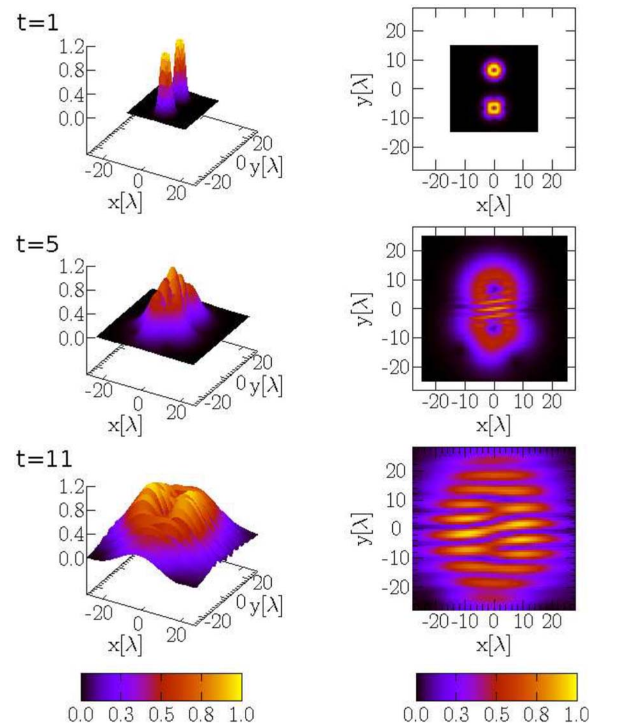

FIG. 4. (Color online) Time evolution of the interference pattern during the overlap of two released condensates initially separated by a distance $d=15\lambda$. Initially each condensate contains $N=6$ atoms and their GS are characterized by $L=6$ at $\Omega/\omega_\perp=0.019$ and by a mixture of $L=6$, 8, and 10 at $\Omega/\omega_\perp=0.0847$, respectively.

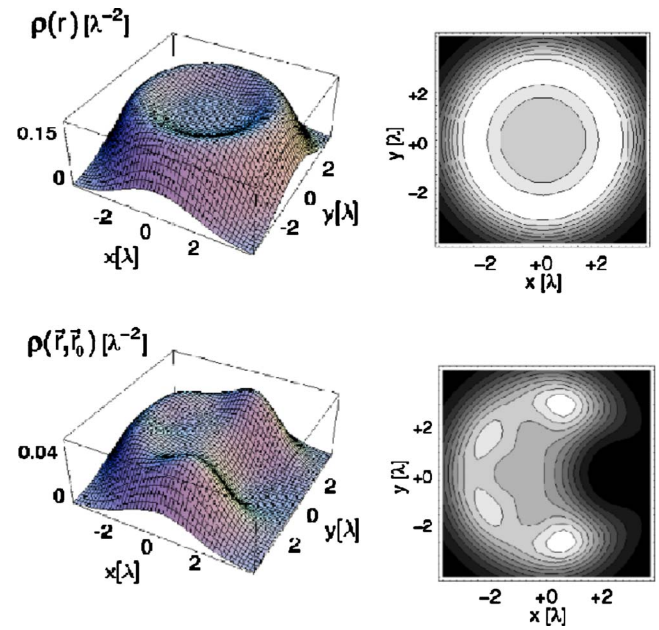

FIG. 5. (Color online) For $N=5$, three-dimensional and contour plots of the density $[\rho(r)]$ and pair correlation function $[\rho(\vec{r},\vec{r}_0)]$ of the Laughlin state ($L=20$).

sibilities not attachable for large systems with thousands of particles, usually used in experiments with rotating BEC. This is the case of a possible test on the meaning of a TOF experiment. We suggest several possibilities for such experiments. Two types of states could be the appropriate candidates: a pure L state with a high degree of condensation at relatively low Ω, and the Laughlin state at Ω close to $\omega_\perp$. In both cases, the density and the pair-correlation function have different profiles in such a way that the outcome of a TOF experiment would distinguish between the two possible meanings of the measurement. As an example, we show in Fig. 5 the outputs of the Laughlin state for $N=5$. One can consider two different possible ways to realize multiple copies of identical small systems. Arrays of rotating microtraps, or lattices with rotating on-site potential wells could be used in one, two, or three dimensions. Alternatively, a one-dimensional lattice rotating around its axis could be employed. One can then, using Mott insulator states, prepare isolated identical states of few atoms. However, the multiple copies of the systems coming from each site of the array and/or lattice, needed to enhance the signal, could, in principle, destroy the patterns. To preserve the pattern structures present in each site, it is necessary to ensure the same orientation (breaking rotational symmetry) and the same global phase (breaking gauge symmetry) of all the copies in the same way. For our first candidate both symmetries must be broken whereas for our second candidate only the first one is needed. The Laughlin state is a unique number state, linear combination of phase states not phase degenerated. To achieve the observability of the pattern, we suggest the following procedure. On the one hand, the presence of a small anisotropy of the trap potential would break rotational symmetry in the same way in each site and, on the other hand, phase correlation could be restored, by embedding the microtraps in a "large" BEC consisting of the same atoms in a different internal state. Such "large" BEC (a "reservoir" with a fixed global phase) would provide the phase-symmetry-breaking mechanism for all small BEC if a weak Raman coupling between the "large" and small system were used [20]. Namely, the measurement, e.g., of unique Laughlin-like states should reveal the Wigner-like structures (according to the n-PCF sampling) if measurement means correlation function and not density. We expect this possibility to happen. Other ways of detecting small rotating clouds of atoms are discussed also by Popp *et al.* in Ref. [21].

As a last comment we want to stress that it is remarkable that even for 5 atoms the density of the Laughlin state (see Fig. 5) at the center, given by $\rho(0)=1.55$, is quite close to its analytical value in the thermodynamic limit, given by $\rho(0)=1/2\pi=1.59$, that is to say, apart from nonessential edge effects, the small systems provide a quite good quatitative picture typical of larger systems with the same filling factor (or density). It gives us confidence in our interpretation of stationary states as vortex states, using the same parameters and definitions used for macrosystems.

IV. CONCLUSIONS

We conclude that we have demonstrated that the use of the eigenfunctions of the OBDM operator provides for relatively low rotation Ω (where condensation is significant and the system is still well inside the LLL regime) a useful and precise tool that complements the analysis of the exact GS obtained from exact diagonalization. This tool is especially useful for vortex states. These eigenfunctions localize and quantize the vortices, and reproduce the time evolution of the interference pattern of two overlapping condensates. We want to stress that our finding is that to a great extent the GS has signatures of a coherent state: on the one hand it reproduces the interference patterns, showing that it has a well defined phase, and, on the other hand, it shows larger fluctuations of n than that of a coherent state with the same expected occupancy. This result suggests even smaller phase fluctuation according to the uncertainty principle. We want to point out that our results imply an alternative interpretation of the subject of the interference pattern formation that has attracted much attention recently. One possibility suggested by Mullin *et al.* [10] is that the experimental measurement projects the initial condensates in Fock states into phase states, the atom distribution between the two components becomes uncertain, and the pattern formation is possible. The other possibility discussed in Ref. [22] is that the interference pattern appears if one includes interaction during the TOF, even for states that initially are Fock states. In our case, the real initial states are Fock states and no interaction is included during the TOF. However, we obtain that the degree of condensation of the initial states is large enough to be properly represented by an OP (condensate WF) with the appropriate qualities of coherence. The mechanism of phase localization is here due to fluctuations of the number of condensed atoms (that jump here and back to the noncondensed cloud) and not the measurement. Indeed, we expect that the exact GS's manifest themselves as phase states even for a small number of particles. In this way the interference pattern is produced and the role of the noncondensed part is to "localize" the phase by allowing fluctuations of n, just smearing the pattern without destroying it. Note, however, that in our picture the process of determination of phase is itself random, and various phases ϕ_k are expected to show up from shot to shot. A deeper analysis of the nature of ψ_1 and its identification as a coherent state is in progress.

ACKNOWLEDGMENTS

We thank J. Dalibard, E. Demler, G. Shlyapnikov, L. Santos, J.M. Pons, and J. Taron for fruitful discussions. We acknowledge financial support of MEC contracts (No. Consolider-Ingenio 2010 "QOIT," No. FIS2004-05639, No. FIS2005-04627, No. 2005SGR00343, and No. 2005SGR00523) and EU IP "Scala."

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