Multipole surface solitons in thermal media

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We address the existence and properties of multipole solitons localized at a thermally insulating interface between uniform or layered thermal media and a linear dielectric. We find that in the case of uniform media, only surface multipoles with less than three poles can be stable. In contrast, we reveal that periodic alternation of the thermo-optic coefficient in layered thermal media makes possible the stabilization of higher-order multipoles. © 2009 Optical Society of America

OCIS codes: 190.0190, 190.6135.

Under appropriate conditions the nonlinear response of suitable materials can be strongly nonlocal, when material regions located far away from the laser beam waist are affected by the beam and contribute to the refractive index [1]. This may occur in materials exhibiting long-range electrostatic interactions, enhanced diffusion, transport processes, etc. For example, nonlocality occurs in liquid crystals, where it affects the interactions between beams [2,3] and results in formation of multipole solitons composed from several out-of-phase spots that can be stable only when the number of spots does not exceed four [4,5]. Thermal media may show even stronger nonlocalities [6–8] because the refractive index shape induced by light extends up to the sample borders, and it is always affected by the boundary conditions [9,10]. These conditions therefore strongly affect propagation of radiation, especially near the material surface. When the conditions for stationary near-surface propagation are met, surface waves form [11,12]. In nonlocal media such waves were studied at the interfaces of diffusive Kerr-type materials [13–15]. Recently fundamental surface waves were observed in uniform focusing thermal media near its thermally insulating boundary attracting light [16]. Such two-dimensional interfaces can also support vortices and dipoles with nodal lines perpendicular to the interface [17]. Surface waves form with defocusing thermal nonlinearities too [18,19]. The properties of one-dimensional multipole surface solitons at thermally insulating interfaces remain unexplored.

In this Letter we reveal that such states possess specific restrictions on the number of poles in stable solitons: only solitons with less than three poles can be stable. Importantly, we also found that if the thermal medium is made of layers with opposite signs of the thermo-optic coefficient [20], it can support stable surface multipoles of high orders. Light beams propagating at interface of such layered medium induce a strong nonlinear lattice that causes their dramatic reshaping with increase of power in contrast to multipoles at interfaces of uniform media.

To describe the propagation of light beams near the thermally insulating interface of thermal medium we use a nonlinear Schrödinger equation for the dimensionless amplitude q of the light field coupled to the equation for normalized temperature variation T,

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - \sigma(\eta)Tq, \quad \frac{\partial^2 T}{\partial \eta^2} = -|q|^2 \quad \text{for} - L \leq \eta \leq 0,$$

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} \text{ for } \eta < -L, \quad \eta > 0. \tag{1}$$

Here η is the transverse coordinate; L is the width of the thermal medium; the parameter $\sigma(\eta) = \sigma_b - \sigma_a \text{sgn}[\sin(\pi\eta/d)]$ accounts for the variations of the thermo-optic coefficient between layers of width d; σ_a, σ_b are the amplitude of variation and constant part of the normalized thermo-optic coefficient; and $\sigma(\eta) > 0$ corresponds to focusing, while $\sigma(\eta) < 0$ corresponds to defocusing nonlinearity. We assume that the refractive index of the surrounding linear medium is equal to the unperturbed refractive index of the thermal material. Thus we solve Eq. (1) with the boundary conditions $T|_{\eta=-L} = 0$ and $\partial T/\partial \eta|_{\eta=0} = 0$, i.e., the $\eta = 0$ boundary is assumed to be thermally insulating (for example, owing to large difference of thermal conductivities of contacting materials), while the $\eta = -L$ boundary is thermally stabilized by means of an external heat sink. We set $L = 25.6$ and $d = 0.5$, which is typical for experiments.

The soliton solutions (including perturbed ones) of Eq. (1) can be written as $q = (w + u + iv)\exp(ib\xi)$, where $w(\eta)$ is a stationary soliton solution, $u(\eta, \xi)$ and $v(\eta, \xi)$ are small perturbations that can grow with complex rate $\delta = \delta_r + i\delta_i$ upon propagation, and b is the propagation constant. Substitution of the light field in such form into Eq. (1) gives in leading order the equation for the stationary profile w, while in the next order one gets the linear eigenvalue problem

$$\delta u = -\frac{1}{2}\frac{d^2 v}{d\eta^2} - \sigma Tv + bv,$$

$$\delta v = \frac{1}{2}\frac{d^2 u}{d\eta^2} + \sigma Tu + \sigma\Delta Tw - bu, \tag{2}$$

which holds for $-L \leq \eta \leq 0$, while outside this region, for $\eta < -L$ and $\eta > 0$, one has $\sigma = 0$. The temperature perturbation stands for $\Delta T = -2\int_{-L}^{0}G(\eta,\lambda)w(\lambda)u(\lambda)d\lambda$,

with the response function $G(\eta,\lambda)=-(\eta+L)$ for $\eta\leqslant\lambda$ and $G(\eta,\lambda)=-(\lambda+L)$ for $\eta\geqslant\lambda$.

First, we consider the properties of multipoles at the edge of uniform thermal media when $\sigma_a=0$, $\sigma_b=1$. Representative shapes of such solitons are shown in Fig. 1. Such solitons reside mostly inside the thermal medium and only weakly penetrate into the linear region. The refractive index profile σT depicted in Fig. 1(d) explains the formation of a thermal lens that results in the concentration of light in the vicinity of the interface. Note that the refractive index is nonzero in the entire thermal medium and decreases almost linearly toward its left border. In all multipoles in uniform media, the pole located farthest from the interface always exhibits a higher amplitude. The higher the number of poles the larger the power $U=\int_{-\infty}^{\infty}|q|^2 d\eta$ carried by solitons at a fixed b. For lowest-order surface states, the power increases almost linearly with b [Fig. 2(a)]. The refractive index distribution broadens with the increase of the number of poles n. Increasing b results in the simultaneous contraction of all poles toward the interface and in the increase in amplitude and peak refractive index at the interface [Figs. 1(a)–1(c)].

One of the central results of this Letter is that only multipoles with the number of poles $n\leqslant 2$ were found to be stable in the case of thermally insulating interface in uniform media. This result resembles the constraint $n\leqslant 4$ on the number of poles in stable multipoles in natural nonlocal materials found in [5]. The presence of the interface results in a more dramatic restriction on a number of spots that can be packed together into stable states. In the case of surface solitons with $n>2$ the growth rate increases monotonically with b, at least far from the cutoff where $\delta_r(b)$ may exhibit a nonmonotonic behavior [Fig. 2(b)]. Also, δ_r increases with n at a fixed b.

Fig. 2. (a) Energy flow versus b for fundamental surface soliton. Circles correspond to profiles shown in Fig. 1(a). (b) δ_r versus b for higher-order solitons with a different number of poles. In all cases $\sigma_a=0$, $\sigma_b=1$.

Importantly, a periodic variation of the thermo-optic coefficient was found to dramatically modify the properties of surface solitons. Here we consider the case of alternating focusing and defocusing layers, i.e., $\sigma_a=1$, $\sigma_b=0$. A light beam entering such a medium self-induces a nonlinear lattice that becomes more and more pronounced with increasing peak amplitude, and that is strongly asymmetric because of the boundary conditions [Fig. 3(d)]. In that case the soliton peak may be localized in any of the focusing layers. Fundamental surface solitons residing in the first layer exhibit pronounced oscillations on their left wings [Fig. 3(a)]. Increasing the peak amplitude results in the concentration of light almost within a single surface layer. Multipoles with $n>1$ centered at intermediate and high powers in the second, third, etc., focusing layers carry the number of poles equal to the number of layer where the most pronounced peak is located [Figs. 3(b) and 3(c)]. The pole amplitudes decrease toward the interface much faster than in uniform medium. All multipoles feature strong in-phase oscillations on their left wings. The power is a
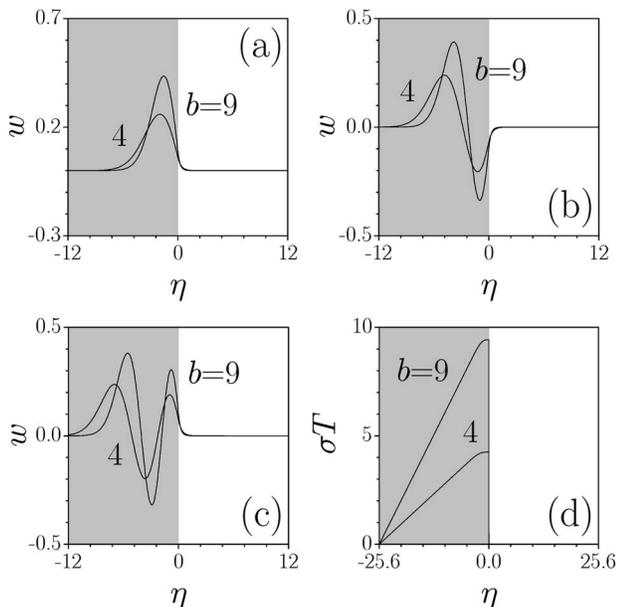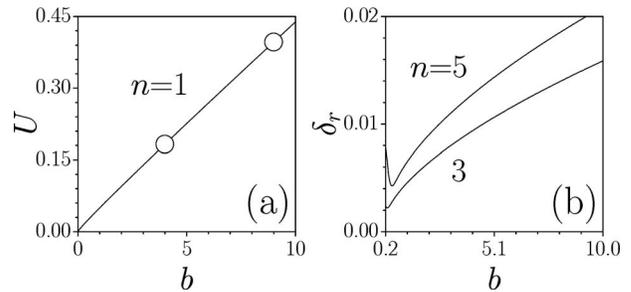

Fig. 1. Profiles of (a) fundamental, (b) dipole, and (c) tripole solitons, and (d) refractive index distributions for fundamental solitons with different b values. In all cases $\sigma_a=0$, $\sigma_b=1$. In gray regions $\sigma>0$.
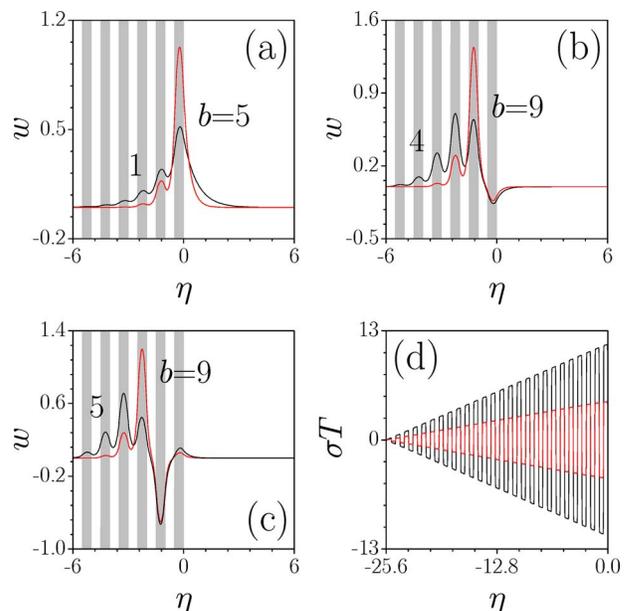

Fig. 3. (Color online) Profiles of (a) fundamental, (b) dipole, and (c) tripole solitons with different b values, and (d) refractive index distributions for fundamental solitons with $b=5$ (black curve) and $b=1$ (gray curve). In all cases $\sigma_a=1$, $\sigma_b=0$. In gray regions $\sigma>0$.
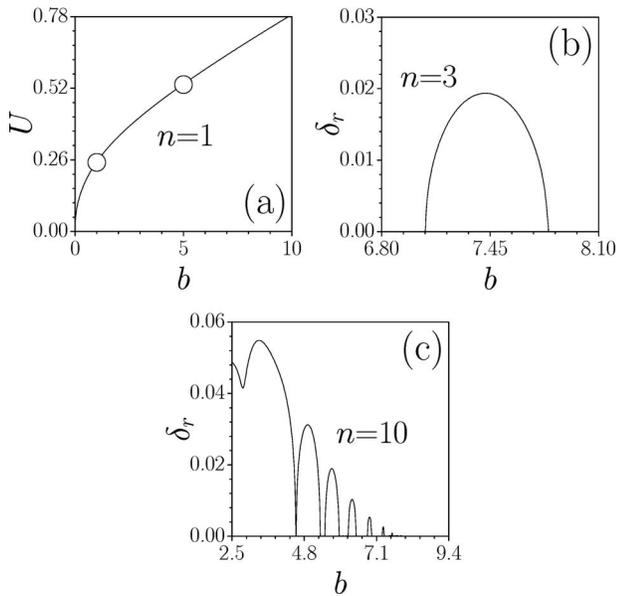
Fig. 4.   (a) Energy flow versus b for fundamental surface soliton. Circles correspond to profiles shown in Fig. 3(a). δ_r versus b for solitons with (b) $n=3$ and (c) $n=10$. In all cases $\sigma_a=1$, $\sigma_b=0$.

monotonically increasing function of b [Fig. 4(a)]. When power decreases, the left outermost pole shifts into the bulk of the thermal medium, gradually jumping between neighboring focusing layers [Fig. 3(c)]. In contrast, when power increases light tends to concentrate almost within a single layer with number n, i.e., out-of-phase oscillations in layers $n-1, n-2, \ldots$ also gradually vanish, and one gets the transformation of multipole soliton into a fundamental-like-looking state with only small wings. Interestingly, the envelope of the refractive index σT for such solitons exhibit almost flat plateau between the layer where soliton is located and interface.

The periodic modulation of the thermo-optic coefficient dramatically modifies the stability of the multipole surface solitons. In such periodic media multipoles with a higher number of spots were found to be stable, provided that the light power exceeds a threshold value. Thus, tripole solitons that were unstable in the uniform thermal medium, in the layered medium exhibit only a single narrow instability domain [Fig. 4(b)]. This is the case for all multipoles
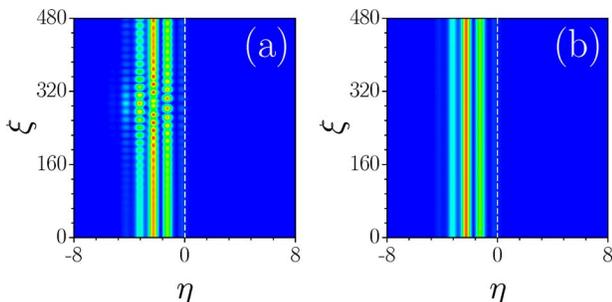

Fig. 5.   (Color online) Propagation of perturbed tripole solitons with (a) $b=7.4$ and (b) $b=8.0$. In both cases $\sigma_a=1$, $\sigma_b=0$. The white dashed line indicates interface position.

with n up to 10 that we considered here. Notice that the complexity of the structure of the instability domain, which for solitons with a large number of spots is usually located close to low-power cutoff, increases with a larger number of poles [see Fig. 4(c) showing instability domains for soliton with $n=10$]. These results are confirmed by direct simulations of propagation of perturbed multipoles in Eq. (1). Figure 5 shows examples of the evolution of unstable [Fig. 5(a)] and stable [Fig. 5(b)] tripole solitons. Development of instability causes pronounced amplitude oscillations and transformation of the internal structure of unstable soliton, but since thermally insulating interface tends to attract light, there is almost no radiation emission. Perturbed stable multipole propagate without any noticeable shape deformations.

Summarizing, while a thermally insulating edge of a uniform thermal medium cannot support stable solitons with more than two poles, we found that one can make stable higher-order multipoles in layered thermal media.

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